

A Fractal Analysis on the Population Distribution of Bukidnon

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Abstract

This paper described and analyzed the population distribution of Bukidnon. A fractal analysis was used by computing the fractal dimensions (λ) of the population distribution of the province. Data are based on the population of 20 municipalities and two (2) cities in 16 years, from 1995 to 2010. Result showed that there is a slight decrease of fractal dimensions which implies that there is an approximately equal population in the municipalities over a period. Various conditions account for the decrease of variability in the population distribution. It may be attributed to their economic situations and cultural practices.

Keywords: Fractal analyses, population distribution, Bukidnon, municipalities, cities

Introduction

Population in Bukidnon has grown rapidly. This means that the population of the municipalities and cities composing the province has also grown. Many researchers have conducted studies on factors influencing the population growth; however, the examination of the variability in the population has been left behind. When we study the variability of the population we examine the population distribution in a given province specifically looking into the pattern as to where people tend to live. To obtain this pattern, a fractal analysis is carried out to describe this pattern.

Hyslip, Trosino and Selig (2002) wrote that fractal analysis is used to characterize irregular patterns and to quantify patterns that are seemingly chaotic and random. The fractal dimension of the pattern varies depending on the degree of roughness of the pattern, and will have a different value for each pattern type, with a fractal dimension being specific for that pattern. Population exhibits are irregular and rough characteristics. Therefore, fractal

analysis can be used precisely to characterize the population. Fractal statistic was used in the study since the fractal dimension was not picture-dependent. It was based on the data.

This paper presents results of a study conducted to explore the use of fractal analysis of population data of the municipalities and cities composing the province of Bukidnon. This paper focused on fractal dimension (λ) and the use of spectrum to determine the variability of the population and identify the different scales of the municipalities in Bukidnon.

Fractal Analysis

According to Mandelbrot (1997), fractal analysis is the process used to determine the fractal dimension to characterize those patterns within nature that are irregular, chaotic or fragmented. It cannot be effectively quantified using classical geometry of whole number dimensions (as cited in Hyslip, 2002). There are various techniques for determining the

fractal dimension of rough patterns. In this study, the λ will be used to determine the fractal dimension on the population distribution of Bukidnon.

Fractal analysis assesses fractal characteristics of data. It consists of several methods to assign a fractal dimension and other fractal characteristics to a data set (Peters, 1996). Fractal analysis is now widely used in all areas of science. An important limitation of fractal analysis is that arriving at an empirically determined fractal dimension does not necessarily prove that the pattern is fractal; rather, other essential characteristics have to be considered (Liu, Zhang, & Yue, 2003). If the fractal dimension is concerned with self-similarity and roughness of geometric objects, it is fractal geometry.

Fractal Statistics, on the other hand, is concerned with data irregularities repeated at different scales generalizing the concept of variances. When variances are too large such that the coefficient of variation, CV, is greater or equal to 1 and when data of lower values are more probable than higher values, it defines fractals, hence, it can be analyzed using fractal statistics. A random observation is said to behave in a fractal distribution if it obeys a power law (Padua, 2014).

Fractal Analysis of Population

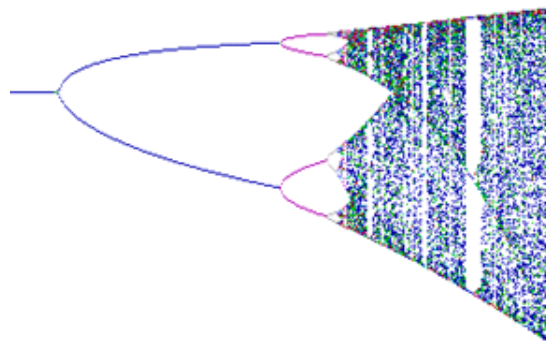
To illustrate the idea of different scales or orders of roughness in the data, different figures were presented. A corresponding spectrum was showed with the different scales of all the cities and municipalities in Bukidnon in terms of its population sizes. The period of the collected data started from year 1995 up to 2010. The fractal dimension (λ) was also computed for each year.

“Thomas Malthus came up with the theory in which he said that the population increases at certain amount of times depending on the growth rate. According to

this theory, the population will increase infinitely. However, the population is really limited by natural resources, such as space and food. Assuming that the maximum possible population the environment can hold is 1, so P is a number from 0 to 1. As the population gets closer to 1, the growth rate is going to decrease and get close to 0. This can be achieved by multiplying the growth rate by $(1-P)$. This way, as P is getting closer to 1, the growth rate will be multiplied by a number that is getting close to 0. Thus, the formula for the growth rate will be

$$\text{new } P = (1 + r) \cdot P - rP^2$$

Knowing this formula, it is easy to determine what the population becomes after a long period of time. As r increases, the number of these values doubles. We call this bifurcation. The graph below has the values of the population for all values of r from 1.9 to 3.



The figure above is identical to the Feigenbaum Fractal. Indeed, we can unexpectedly find fractals even in the growth of population”

Conceptual Framework

The spread of the population of people in a given province is influenced by many factors.

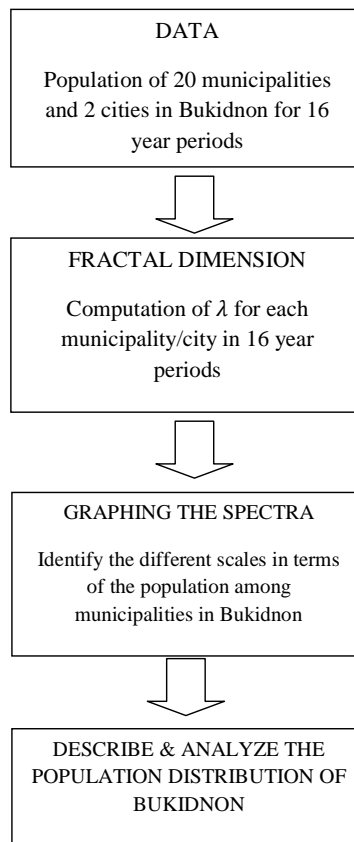


Figure 1. Conceptual paradigm of the present study.

It is an important aspect in identifying the lift of the economy. The pattern where people tend to live can be better understood by studying the variability of the population sizes in a given province. This study focuses mainly on the description and analysis of the population distribution in the province of Bukidnon.

Objectives

This study described and analyzed the population distribution of Bukidnon. The following objectives are specified:

1. To describe the population distribution of Bukidnon using fractal dimension;
2. To identify the different scales among municipalities and cities in terms of its population; and
3. To identify the factors influencing the population distribution of Bukidnon.

Methodology

Bukidnon is a non-coastline plateau in North Central Mindanao. It is composed of 20 municipalities and two (2) cities.

Table 1
Municipalities and Cities of Bukidnon, Date of Creation, Area and Income Classification

City/Municipality	Date of Creation	Area (km ²)	Income Classification (DOF)
Baungon	July 1, 1956	175.86	2nd Class Municipality
Cabanglasan	August 13, 1979	209.00	3rd Class Municipality
Damulog	August 16, 1971	245.66	4th Class Municipality
Dangcagan	August 29, 1961	115.15	3rd Class Municipality
Don Carlos	June 18, 1966	157.02	1st Class Municipality
Impasug-ong	September 1, 1914	1,071.67	1st Class Municipality
Kadingilan	August 16, 1971	172.06	3rd Class Municipality
Kalilangan	June 18, 1966	153.59	2nd Class Municipality
Kibawe	July 1, 1956	214.35	2nd Class Municipality
Kitaotao	June 18, 1966	150.74	1st Class Municipality
Lantapan	June 18, 1966	240.76	1st Class Municipality
Libona	July 1, 1956	244.95	1st Class Municipality
Malaybalay City	March 22, 1998	984.38	1st Class Component City
Malitbog	June 25, 1963	260.53	2nd Class Municipality
Manolo Fortich	June 21, 1957	506.64	1st Class Municipality
Maramag	July 1, 1956	351.72	1st Class Municipality
Pangantucan	June 25, 1963	343.34	1st Class Municipality
Quezon	June 18, 1966	409.41	1st Class Municipality
San Fernando	June 18, 1966	638.63	1st Class Municipality
Sumilao	July 1, 1956	207.49	4th Class Municipality
Talakag	1917	833.70	1st Class Municipality
Valencia City	January 12, 2001	607.13	2nd Class Component City

Population of each municipality and city of Bukidnon was studied through 16 time periods. The population data were obtained from the National Statistics Office (NSO) from 1995 to 2010.

The fractal distribution was fitted to data by arranging the data such that $x_1 < x_2 < \dots < x_k < \dots < x_n$. After such, scale and lambda were computed.

Fractal dimension (λ) for 16 time periods for the population of Bukidnon was computed using the formula:.

$$\hat{\lambda} = 1 + n \left[\sum_{i=1}^n \log \frac{x_i}{\theta} \right]^{-1}$$

where $\theta \leq x_i$ and θ is the smallest observation.

The fractal dimension can be thought of as an indicator of the variability of population sizes among the municipalities of Bukidnon. If the fractal dimension is within the range of $0.6 < \lambda < 3.4$, a fractal analysis can be applied.

To identify which one of the municipalities belongs to the same scale and which has the highest fractal dimension (λ), a fractal spectrum was plotted. Every five years, from 1995 to 2010, a spectrum was plotted. The number of elbow of the spectrum corresponds

to the number of scales. As the population gets larger, the scale will get smaller and vice versa. This relationship can be shown in the formula below:

$$s = \frac{1}{\log(\frac{X_i}{\theta})}$$

where, s is scale, θ is the smallest observation. If we have the scale, we obtain

$$\lambda_{(\alpha)} = 1 - \log(1 - \alpha)S$$

The $\lambda_{(\alpha)}$ can be the indicator of the population size variability of the given municipality or city.

Results and Discussion

Table 3 shows the fractal dimension and the Coefficient of Variation in the period of 15 years (1995 – 2010).

Table 3
Dimensions and Coefficient of Variation from 1995 to 2010

Year	Fractal Dimension	CV
1995	2.16	0.703
1996	2.15	0.706
1997	2.15	0.710
1998	2.14	0.713
1999	2.13	0.716
2000	2.13	0.719
2001	2.12	0.723
2002	2.11	0.726
2003	2.10	0.730
2004	2.10	0.733
2005	2.09	0.737
2006	2.09	0.741
2007	2.08	0.744
2008	2.07	0.749
2009	2.07	0.754
2010	2.06	0.756

A fractal analysis can be used to analyze the population distribution in the province of Bukidnon, since the computed fractal dimensions from 1995 to 2010 are within the range, $0.6 \leq \lambda \leq 3.4$. On the other hand, the ruggedness of the population is bounded since the coefficients of variations are less than one.

Based on Table 3, there is a decrease of fractal dimensions which means that there is a decrease in the population size variability among towns in the province. There are years which have the same lambda, which implies that there is not so much difference in their population during these periods.

Specifically, a spectrum was plotted to identify which among the municipalities belong to the same scale and to inspect its individual population size variability.

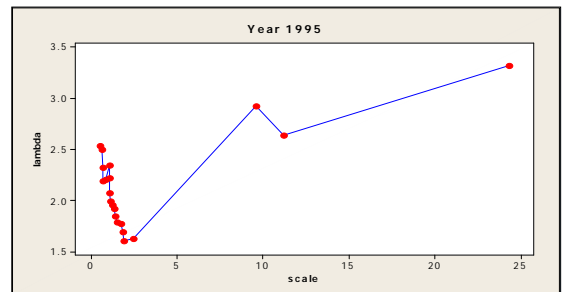


Figure 2. Spectrum of the scale vs. lambda in the year 1995.

It can be shown in Figure 2 that in the year 1995 there are six (6) scales. The towns are scaled according to their population sizes. Table 3 shows the municipalities composing the different scales.

Table 3
Scale and Lambda of the Municipalities/
Cities in Bukidnon in 1995

Scale 1			Scale 4		
	Scale	lambda		Scale	lambda
Malaybalay	0.50	2.54	Cabanglasan	1.50	1.79
Quezon	0.63	2.50	Kahilangan	1.71	1.77
ManoloFortich	0.67	2.33	Kadingilan	1.81	1.69
Maramag	0.70	2.19	Impasug-ong	1.90	1.61
			Baungon	2.44	1.63
Scale 2			Scale 5		
	Scale	lambda		Scale	lambda
Don Carlos	0.82	2.21	Dangcagan	9.59	2.92
Talakag	1.04	2.35	Malitbog	11.18	2.64
Pangantucan	1.06	2.22			
Scale 3			Scale 6		
	Scale	lambda		Scale	lambda
Kitaotao	1.06	2.08	Sumilao	24.32	3.32
Lantapan	1.11	1.99			
San Fernando	1.21	1.95			
Libona	1.33	1.92			
Kibawe	1.39	1.84			

The first scale (scale 1) is composed of the towns with the largest population in the province. The order of the scales is from the municipalities with the highest population down to the municipalities with the smallest population. The city of Valencia has the highest population followed by Malaybalay, Quezon, Manolo Fortich and Maramag, while the municipality of Sumilao has the smallest population. It can be supported in the result of Census and Housing, Population (2000), where Valencia was the largest among the 22 municipalities in Bukidnon in terms of population (147,924 persons or 13.95 percent of the total population of the province). It was followed by Malaybalay, the provincial capital (11.66 percent), Quezon (7.79 percent), Maramag (7.09 percent), and Manolo Fortich (7.00 percent). On the other hand, Sumilao had the smallest population with 1.61 percent (Census and Housing, 2000).

The values of lambda for all municipalities are more or less equal except for the Municipality of Sumilao with the highest lambda. It shows that it has the highest variability in terms of the population size compared to the other municipalities in the province. This implies that all municipalities have approximately equal population except for

Sumilao which has a wide gap of population among other municipalities in Bukidnon. This gap in population can somehow be attributed by its income classification. "It is a fourth class municipality in the province of Bukidnon, it has also the smallest population in the province from the year 1918 to year 1995. Its population sharply decreased from 2,670 in 1939 to 1,981 as an adverse effect of World War II up to 2000" (Sumilao Profile, n.d.).

However, in 2000 there was a rapid growth of population in Sumilao. According to Sumilao profile (2000), its population had increased to 17,958, the second lowest in the province and a growth rate of three (3) percent. The increase in population could partly be attributed to immigration of agro-industrial workers attracted by the economic activities in the municipality brought by multi-national companies and big-time investors" (Sumilao Profile, n.d.). It can be shown in Table 4 that Sumilao has increased in population, leaving behind the municipality of Malitbog.

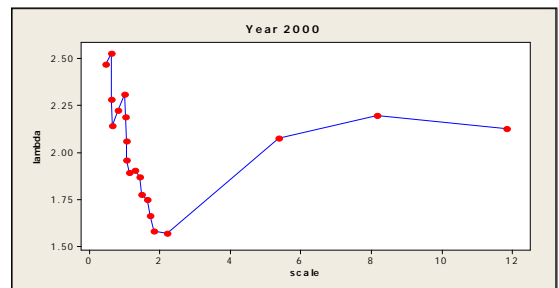


Figure 3. Spectrum of the scale vs. lambda in the year 2000.

Based on Figure 3, there is a decrease in the scale of the municipalities in the province of Bukidnon. The decrease of lambda in Table 4 is reflected in the decrease of scales of the municipalities.

Below is the table showing the municipalities belonging to their respective scales.

Table 4
*Scale and Lambda of the Municipalities/
 Cities in Bukidnon in 2000*

Scale 1		
	Scale	lambda
Malaybalay	0.475	2.468
Quezon	0.637	2.527
ManoloFortich	0.644	2.283
Maramag	0.670	2.142
Don Carlos	0.825	2.222

Scale 2		
	Scale	lambda
Talakag	1.007	2.308
Kitaotao	1.036	2.187
Lantapan	1.048	2.060
Pangantucan	1.072	1.958
San Fernando	1.134	1.894

Scale 3		
	Scale	lambda
Libona	1.303	1.903
Cabanglasan	1.435	1.870
Kibawe	1.479	1.778
Kalilangan	1.655	1.748

Scale 4		
	Scale	lambda
Impasug-ong	1.732	1.663
Kadingilan	1.831	1.583
Baungon	2.216	1.571

Scale 5		
	Scale	lambda
Dangcagan	5.365	2.077
Sumilao	8.160	2.196
Malitbog	11.830	2.128

Table 4 shows the groups of municipalities with almost equal population sizes. The four (4) municipalities which include Malaybalay, Quezon, Manolo Fortich and Maramag have been consistent with the group of towns with the highest population. On the other hand, the Municipality of Malitbog has now the smallest population.

According to the census conducted by the National Statistics Office (2000), Sumilao ranks the highest percent increase at 18.90% with an increase population of 2,146 between 1990 and 1995. The population of Sumilao increased to 17,958 brought about by in-migration of agro-industrial workers from the neighboring towns of Manolo Fortich, Impasugong, Malaybalay City and from the provinces of Misamis Oriental, Misamis Occidental, Camiguin, Bohol and other places in Luzon. The approximately equal population of these towns can be attributed by the movement of people into these places.

It can be shown in Table 4 that there is a more equal population among the municipalities in the province compared to 1995. The highest population size variability is the Municipality of Quezon which implies that it has a rapid increase in terms of population. Aside from the fact that Quezon belongs to a first class

municipality, there are many industries that have high potential in the municipality, i.e., alcohol manufacturing, rum manufacturing, grains processing, feed milling, meat processing, coconut processing, fruit and food processing, mineral water processing, banking services, and commercial fertilizer processing (Quezon, Bukidnon, n.d.).

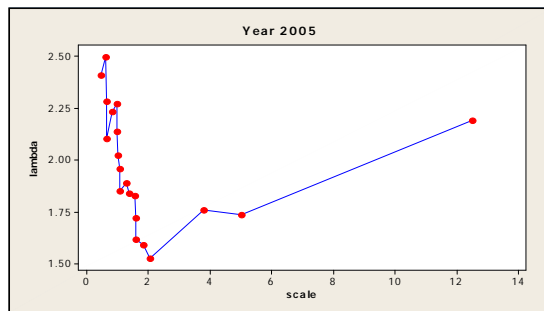


Figure 4. Spectrum of the scale vs. lambda in the year 2005.

Based on Figure 4, there are five (5) scales grouping the municipalities in the province of Bukidnon. There is no increase in the number of scales reflected in the spectrum. This shows that there is no increase in the population size variability among municipalities in the province which implies that there is no increase or decrease of population among towns; hence, there is no in-migration or out-migration in the province during the year 2005.

Table 5 shows the towns composing the respective scales. The first scale (scale 1) is still composed of the four towns, namely, Malaybalay, Manolo Fortich, Maramag and Quezon. However, Manolo Fortich is now the second largest population leaving behind Quezon. The increase in the population of Maramag pushed the municipality to a third place. Among the two cities and 20 municipalities comprising the province of Bukidnon, the City of Valencia had the largest population with a share making up 14.0 percent of the provincial population. According to

Census of Population and Housing (2010), the City of Malaybalay, the provincial capital, was second with 11.8 percent share, followed by the municipalities of Quezon with 7.3 percent, Manolo Fortich and Maramag with 7.0 percent each, and Talakag with 5.2 percent.

Table 5
Scale and Lambda of the Municipalities/Cities in Bukidnon in 2005

Scale 1		
	Scale	lambda
Malaybalay	0.456	2.408
ManoloFortich	0.624	2.497
Maramag	0.644	2.284
Quezon	0.648	2.104

Scale 2		
	Scale	lambda
Don Carlos	0.833	2.234
Talakag	0.980	2.273
Lantapan	0.994	2.139
Kitaotao	1.011	2.023
San Fernando	1.070	1.956
Pangantucan	1.080	1.852

Scale 3		
	Scale	lambda
Libona	1.282	1.889
Cabanglasan	1.382	1.838
Kibawe	1.571	1.826
Impasug-ong	1.596	1.721
Kalilangan	1.610	1.617
Kadingilan	1.853	1.590
Baungon	2.040	1.526

Scale 4		
	Scale	lambda
Dangcagan	3.787	1.760
Sumilao	5.012	1.735

Scale 5		
	Scale	lambda
Malitbog	12.510	2.192

The increase of population in the municipalities of Lantapan, San Fernando and Impasug-ong in 2005 shown in Table 5 elevated these municipalities to higher scales because of the growing economic opportunities in the said municipalities.

Accordingly, “in Lantapan, there are 428 commercial establishments registered at the Licensing Division of the municipality, 15 of which are classified under agro-industrial, manufacturing and servicing. Trading activities include wholesale buying of corn, coffee and vegetables by traders from within Lantapan, neighboring towns in Bukidnon and Cagayan de Oro City” (Lantapan, n.d.).

In San Fernando, “the municipality is predominantly agricultural with corn as the major product followed by rice and abaca, rattan, banana, bamboo, vegetables. Root crops are also produced in the municipality” (San Fernando, Bukidnon, n.d.).

In Impasug-ong, “there are nine (9) commercial cattle farms in the area holding

1,143 heads in all. The LGU also manages a communal ranch in a 649-hectare land that holds 300 heads of cattle. This supports the dispersal and breed improvement program of the municipality. There are also 28 commercial poultry breeder farms in the municipality with an estimated combined population of 775,000 heads” (Impasug-ong, Bukidnon, n.d.).

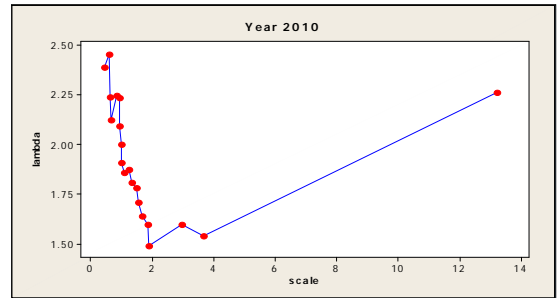


Figure 5. Spectrum of the scale vs. lambda in the year 2010.

Figure 5 shows that the spectrum is divided into five (5) scales. It can be shown that there is no increase in the number of scales from 2000 up to 2010 which implies that the order of municipalities in terms of population size is similar to the previous year.

The scales of the municipalities composing the province are shown in the table below.

Table 6
Scale and Lambda of the Municipalities/Cities in Bukidnon in 2010

Scale 1		
	scale	lambda
Valencia	0.450	2.390
ManoloFortich	0.607	2.455
Maramag	0.622	2.239
Quezon	0.658	2.122

Scale 2		
	scale	lambda
Don Carlos	0.841	2.246
Lantapan	0.949	2.233
Talakag	0.956	2.094
Kitaotao	0.989	2.000
San Fernando	1.016	1.908
Pangantucan	1.087	1.857

Scale 3		
	scale	lambda
Libona	1.263	1.875
Cabanglasan	1.336	1.810
Impasug-ong	1.486	1.782
Kalilangan	1.569	1.709
Kibawe	1.669	1.639

Scale 4		
	scale	lambda
Kadingilan	1.873	1.597
Baungon	1.898	1.489

Scale 5		
	scale	lambda
Dangcagan	2.965	1.595
Sumilao	3.675	1.539
Malitbog	13.226	2.261

Based on Table 6, the City of Valencia has the highest population which belongs to the first scale (scale 1). There is also an increase in the population of Lantapan which makes it in the sixth place and belongs to the second scale. The Municipality of Sumilao has returned to the last scale (scale 7) which shows that there is a decrease in its population.

Table 6 provides the increase of variability of Malitbog, with the value that is almost equal to the higher scale municipalities. This implies that there is a rapid increase in the population of Malitbog. "Based on the number of its inhabitants, Malitbog is number 1135 in the most populous cities in the Philippines and at 329 in the Mindanao group of islands and at 19th of the most populous municipalities of the province of Bukidnon" (Malitbog, Bukidnon, n.d.). The main livelihood of Malitbog is agriculture. Accordingly, "Malitbog is home to a number of residents most of whom are actively engaged in agriculture. Despite its remote topographical location, the people of Malitbog, particularly the Local Government Unit of Malitbog, has always been receptive to change and development. It has opened its doors to new ideas, strategies, and systems of the government" (Malitbog, Bukidnon, n.d.). A good bumper crop in agriculture could attract the in-migration in the said municipality.

According to history, "The pioneers of the municipality were a group of Bukidnon tribe" (Malitbog, Bukidnon, n.d.). According to Cabriles (1981), marriage among the Bukidnons is either endogenous (marriage taking place within one's group) or exogenous (marriage taking place outside of one's group) and is made through parental arrangement. Polygamy is also practiced but is not very common among the affluent. These are factors that would influence the increasing population.

The increase of population in the municipality can also be contributed by the early marriage of the settlers. According to Cabriles

(1981), an important factor which affects fertility level is age at first marriage. It has been generally observed that the younger the age of marriage, the greater the exposure to the risk of childbearing, thus, the higher the probability of having children. It can be supported that majority of the population of Malitbog belong to the younger group. "The younger age group is higher compared to the older population age group in Malitbog" (Malitbog, Bukidnon, n.d.)

Conclusion

The fractal dimension was used as indicator in the variability of the population. The differences of the variability among towns measure the equality of the municipalities' population sizes. The approximately equal variability implies that there is an approximately equal population distribution of the municipalities in the province.

For a period of 16 years, there is a decrease of fractal dimension (see Table 3) in population distribution of Bukidnon which implies that the populations of the municipalities have become approximately equal despite its differences in the areas and the income classifications (see Table 1). Several factors have contributed to the equality of their population. One of these factors is the economic activities of each municipality. The rapid growth of population in small scale municipalities such as Sumilao, San Fernando, Lantapan, and Impasug-ong is due to its growing economic opportunities which led its population to be almost equal to the large scale municipalities. However, the increase of population in Malitbog, is brought about by the settlers' practices.

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