Detailing Poverty Incidence through Fractals: Which of the Gross National Product or Multidimensional Poverty Index Explain Poverty Incidence Better?

Joy M. Mirasol, Zeny L. Maureal and Noel B. Lacpao

Abstract

This paper attempts to explain poverty incidence of the 97 countries using fractal analysis. Gross National Product (GNP) and Multidimensional Poverty Index (MPI) of each country were used as poverty indicators. Fractal dimensions were obtained, compared and analyzed. The three variables have fractal characteristics of ruggedness and self-similarity. Results revealed that the ruggedness of poverty incidence across the countries is due to the ruggedness of the MPI, that is, the deprivation to basic services such as health, education and standard of living affects the quality of living. Thus, MPIs explain poverty incidence more precisely. With this finding, implications to policymakers to alleviate poverty can be addressed.

Keywords: Fractal analysis, Gross National Product, Multidimensional Poverty Index (MPI), poverty indicators, spectrum

Introduction

Poverty is a global issue. What causes poverty and solution to alleviate deprivations remain a huge concern not only in the Philippines but all over the world. About half of the world (over three billion) people live less than $2.50 a day (Shah, 2013). Poverty and its consequences are of central importance to public policy makers. However, the cause of poverty remains an elusive problem.

Attempts have been made to explain the complexities of poverty through different mathematical models. Chattopadhyayet (2006) made a theoretical analysis on the changes in poverty with respect to the 'global' mean and variance of the income distribution using Indian survey data. When income obeys a log-normal distribution, a rising mean income indicates a reduction in poverty while an increase in the variance of the income distribution increases poverty. This altruistic view for a developing economy, however, is acceptable once the poverty index is found to follow a pareto distribution. Here although a rising mean income indicates reduction in poverty due to the presence of an inflexion point in the poverty function, there is a critical value of the variance below which poverty decreases with increasing variance; while beyond this value, poverty undergoes a steep increase followed by a decrease with respect to higher variance. Hence, the pareto poverty function satisfies all three standard axioms of a poverty index with inflexion point as the poverty line.

While Coromaldi and Zoli (2012) derived indicators of multiple derivation by applying a particular multivariate statistical technique, the nonlinear component analysis overcomes traditional limit of many of the used methodologies for poverty measurement. Second, on the basis of the aforementioned indicators, they provide an accurate identification of the poor in Italy by analyzing both as a distinct phenomenon of poverty in different life domains and as a single multidimensional concept. The main determinant of poverty in Italy is investigated by estimating logit regressions and an ordered probit model.

Vijayakumarf and Olga (2012) found and analyzed the significant determinant of the incidence of poverty in the estate sector of Sri Lanka where the highest level of chronic poverty and unemployment exist. The Ordinary Least Squares (OLS) regression analysis indicates that variables such as industrial employment, education, access to market and infrastructure significantly and negatively affect the poverty incidence of the estate sector. Agricultural employment has a negative impact but not significant. Analysis with the Durbin-Watson stat confirms that there is no autocorrelation between the variables

Tzavidis and Salvati (2007) used M-quantile models in deriving small area estimates of poverty and inequality. Unlike traditional random effect models, M-quantile models do not depend on strong distributional assumptions and automatically provide outlier robust inference.

In the Philippines, Huelgas (2011) used the classical regression model and the spatial lag model in estimating city and municipal poverty incidence. These models provided estimates for cities and municipalities with no direct estimates as well as to present estimates with improved precision for cities and municipalities with unreliable direct estimates. The study used five factors namely, the employment of the household head, the education of the household male members, age structure of the household members, the materials of which the house is made of and a proxy measure of the community's progress. In classical regression model, results show the importance of education to the members of the household particularly that of male members, and the development and infrastructure of the community where the household resides. Moreover, educational attainment of the household members has the highest effect on poverty. Also, the identity link performs better than the logit or probit link in terms of a lower mean absolute percentage error. In the spatial regression model, the 25-km distance threshold provides the optimum model with relatively lower mean absolute percentage error and root mean square error compared to other threshold distance matrices. Lastly, the spatial lag model using distance matrix was found to be superior than classical regression model by virtue of lower mean absolute percentage error, lower root mean square error and estimates with lower standard errors and coefficients of variation.

In 2010, the Oxford Poverty and Human Development Initiative (OPHI) of Oxford University and the Human Development Report Office of the United Nations Development Programme (UNDP) launched a new poverty measure that gives a multidimensional picture of people living in poverty, the Multidimensional Poverty Index (MPI). MPI identifies deprivations across health, education and living standards and shows the number of people who are multidimensionally poor and the deprivations that they face at the household level. It uses 10 indicators across dimensions. Figure 1 shows the indicators.

A natural phenomenon that demonstrates a repeating pattern is called fractal. Poverty incidence in a large scale exhibits this reiterating form. In this paper, the authors used fractal analysis to determine the root cause of incidence of poverty more precisely. Particularly, this method will assess the fractal characteristics of poverty incidence data. Results of this paper will hopefully serve as basis for policymakers to formulate policies to address poverty incidence.

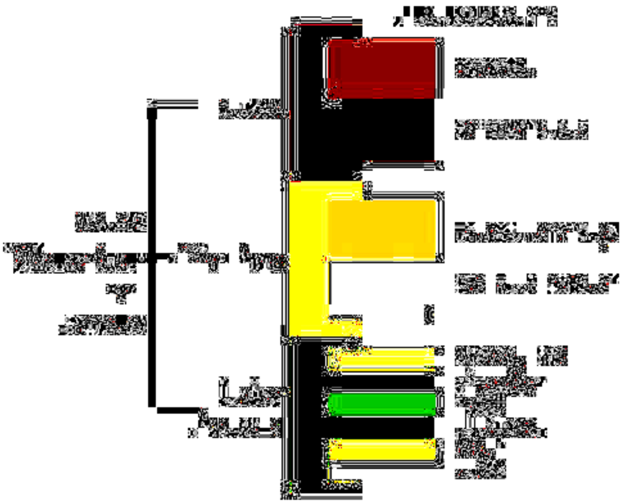

Figure 1. MPI Indicators

Source: Oxford Poverty and Human Development Initiative, 2011 and 2013.

Conceptual Framework

Poverty is a phenomenon that has caught attention of the scientists in the different fields. The complexity of the phenomenon demands a corresponding nonlinear complex and dynamical approach embodied in emerging discipline of fractals.

The incidence of poverty across geographic boundaries can be described as a rugged landscape of poor living among the rich. High poverty incidence is noted when the disparities between income of rich and poor are high. One might therefore surmise that the poverty incidence is closely related to the phenomenon of the distribution of wealth among nations. But some theorists claim that wealth alone cannot explain the rugged environments which host the poor among the rich. This study explores the other factor that could explain the observed irregular and rugged behavior of poverty across the globe.

Objectives

The paper identified the indicators of poverty incidence using fractal analysis. Particularly, this method assessed the fractal characteristics of poverty incidence data.

Basic Concepts

Fractal Statistics is concerned with data irregularities repeated at different scales generalizing the concept of variances. When the variances are too large such that the coefficient of variation (CV) is greater than 1, and if there are more lower values than higher values, then the data are fractals.

Self-Similarity at Various Scales

A function f (x) is self-similar (or homogeneous of degree k) if

$f(bx) = b^k f(x)$, k is any real number. (1)

The only self-similar function in one variable is the monomial function

$f(x) = ax^k$, k is any real number. (2)

From the class of self-similar functions, a subclass that gives larger weights to the lower values is obtained, that is, the function

$f(x) = ax^{-\lambda}$ where $\lambda > 0,\ x > 0$. (3)

Given f (x) in (3), we want to convert this into a probability distribution such that

$$\int_{-\infty}^{\infty} f(x)dx = 1. \qquad (4)$$

Using (4), $\int_0^{\infty} ax^{-\lambda}dx = 1$, solve for a. Then

$$a = \frac{1}{\int_{\theta}^{\infty} x^{-\lambda} dx}$$

$$a = \frac{\lambda - 1}{\theta^{\lambda-1}}, \lambda > 1 \quad (5)$$

Quantitative Model for Fractal Statistics

A random variable x is said to behave in a fractal distribution if it obeys a power-law:

$$f(x) = \frac{\lambda-1}{\theta}\left(\frac{x}{\theta}\right)^{-\lambda}, \lambda > 1 \text{ and}$$
$$\theta = minimum\{x\} \quad (6)$$

where λ is called the fractal dimension of the distribution.

The model in (6) has two parameters, θ and λ, which are both unknown. To estimate the value of θ, we get the minimum of the data, that is,

$$\hat{\theta} = minimum\{x\}. \quad (7)$$

To estimate the value of ë, we use the Maximum Likelihood Estimator (MLE).

Maximum Likelihood Estimator of λ

Given the observations, from the fractal distribution f (x) in (6), the likelihood function L is obtained.

$$L = \prod_{i=1}^{n} f(x_i) = \left(\frac{\lambda-1}{\theta}\right)^n \prod_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{-\lambda} \quad (8)$$

Taking the logarithm of L,

$$logL = nlog\left(\frac{\lambda-1}{\theta}\right) - \lambda \sum_{i=1}^{n} log\left(\frac{x_i}{\theta}\right). \quad (9)$$

The MLE of λ is the value that maximizes the (8) or (9) by taking its derivative with respect to λ and equate to zero:

$$\frac{\partial logL}{\partial \lambda} = 0$$

$$\frac{\partial logL}{\partial \lambda} = n.\frac{1}{\lambda - 1} - \sum \log\left(\frac{x_i}{\theta}\right) = 0$$

Solving for the estimate of λ

$$\hat{\lambda} = 1 + \frac{n}{\sum \log\left(\frac{x_i}{\theta}\right)}. \quad (10)$$

Fitting a Fractal Distribution to Data

Given the observations, $x_1, x_2, x_3, \ldots, x_n$ from the fractal distribution f (x), arrange them such that $x_1 < x_2 < \cdots < x_k < \cdots < x_n$. Weights denoted by α_k are assigned to $x_{(k)}$,

$$\alpha_k = \frac{k}{n}, \quad k = 1, 2, 3, \ldots, n, \quad (11)$$

The αth percentile of the distribution obeys the rule

$$\int_{\theta}^{x_\alpha} f(x)\, dx = \alpha. \quad (12)$$

$$\int_{\theta}^{x_\alpha} \frac{\lambda - 1}{\theta^{\lambda-1}} \left(\frac{x}{\theta}\right)^{-\lambda} dx = \alpha. \quad (13)$$

Solving for λ_α, (13) gives

$$\lambda_\alpha = 1 - \frac{\log_e(1-\alpha)}{log_e\left(\frac{x_\alpha}{\theta}\right)}. \quad (14)$$

Letting, $S = \frac{1}{log_e\left(\frac{x_\alpha}{\theta}\right)}$, then

$$\lambda_\alpha = 1 - ln(1-\alpha)S. \quad (15)$$

S is called the scale of fractal spectrum. The fractal spectrum is the visual and graphical representation of the charges in fractal dimensions as function of scales S.

Table 1

Data Showing the MPIs, PIs and GNPs of the 97 Countries

	Country	MPI	% of People who are Income Poor ($1.25/day)	% of People who are Income Poor ($2.00/day)	GNP
1	Albania	0.005	0.6	4.3	4,090
2	Angola	0.452	54.3	70.2	4,580
3	Argentina	0.011	0.9	2.4	8,450
4	Armenia	0.004	1.3	12.4	3,720
5	Azerbaijan	0.021	1	7.8	6,050
6	Bangladesh	0.292	49.6	81.3	840
7	Belize	0.024	12.4	24.5	4,180
8	Benin	0.412	47.3	75.3	750
9	Bhutan	0.119	26.2	49.5	2,420
10	Bolivia	0.089	13.6	25.1	2,220
11	Bosnia and Herzegovina	0.003	0	0.2	4,650
12	Brazil	0.011	3.8	9.9	11,630
13	Burkina Faso	0.536	56.5	81.2	670
14	Burundi	0.53	81.3	93.5	240
15	Cambodia	0.251	28.3	56.5	880
16	Cameroon	0.287	9.6	30.4	1,170
17	Central African Republic	0.512	62.8	80.1	490
18	Chad	0.344	61.9	83.3	740
19	China	0.056	15.9	36.3	5,740
20	Colombia	0.022	16	27.9	6,990
21	Comoros	0.408	46.1	65	840
22	Cote d'Ivoire	0.353	23.8	46.3	1,220
23	Croatia	0.016	0.1	0.1	13,290
24	Czech Republic	0.01	0.1	0.2	18,130
25	Djibouti	0.139	18.8	41.2	1,280
26	Dominican Republic	0.018	4.3	13.6	5,470
27	Congo, Democratic Republic of	0.393	59.2	79.6	2,550
28	Ecuador	0.009	4.4	13.6	5,190
29	Egypt	0.024	2	18.5	3,000
30	Estonia	0.026	0.5	1.5	15,830
31	Ethiopia	0.562	39	77.6	410
32	Gabon	0.161	4.8	19.6	10,070
33	Gambia	0.324	34.3	56.7	510
34	Georgia	0.003	15.3	32.2	3,280
35	Ghana	0.144	30	53.6	1,550
36	Guatemala	0.127	11.7	24.3	3,120
37	Guinea	0.506	43.3	69.6	460
38	Guyana	0.053	8.7	18	3,410
39	Haiti	0.299	54.9	72.2	760
40	Honduras	0.159	23.3	35.4	2,070
41	Hungary	0.016	0.2	0.4	12,390
42	India	0.283	41.6	75.6	1,530
43	Indonesia	0.095	18.7	50.6	3,420
44	Iraq	0.059	4	25.3	5,870
45	Jordan	0.008	0.4	3.5	4,720
46	Kazakhstan	0.002	0.2	1.5	9,730
47	Kenya	0.229	19.7	39.9	840
48	Kyrgyzstan	0.019	1.9	29.4	990
49	Laos	0.267	33.9	66	1,260
50	Latvia	0.006	0.3	1	14,180

Source: Alkire, Roche, Santos and Seth (2011). Multidimensional Poverty Index: 2011 Data

	Country	MPI	% of People who are Income Poor ($1.25/day)	% of People who are Income Poor ($2.00/day)	GNP
51	Lesotho	0.156	43.4	62.3	1,380
52	Liberia	0.485	83.7	94.8	370
53	Macedonia	0.008	0.3	4.3	4,690
54	Madagascar	0.357	67.8	89.6	430
55	Malawi	0.381	73.9	90.5	320
56	Mali	0.558	51.4	77.1	5,750
57	Mauritania	0.352	21.2	44.1	1,110
58	Mexico	0.015	1.8	8.6	9,740
59	Moldova	0.007	1.9	12.5	2,070
60	Montenegro	0.006	0.1	0.2	6,940
61	Morocco	0.048	2.5	14	2,940
62	Mozambique	0.512	59.6	81.8	510
63	Namibia	0.187	49.1	62.2	5,670
64	Nicaragua	0.128	15.8	31.9	1,650
65	Niger	0.642	43.1	75.9	370
66	Nigeria	0.31	64.4	83.9	1,430
67	Pakistan	0.264	22.6	61	1,260
68	Paraguay	0.064	5.1	13.2	3,290
69	Peru	0.086	5.9	14.7	5,880
70	Philippines	0.064	22.6	45	2,470
71	Congo, Republic of the	0.208	54.1	74.4	2,550
72	Russia	0.005	0	0.1	12,700
73	Rwanda	0.426	76.8	89.6	560
74	São Tomé and Príncipe	0.154	29.7	55.9	1,320
75	Senegal	0.384	33.5	60.4	1,040
76	Serbia	0.003	0.1	0.7	5,280
77	Sierra Leone	0.439	53.4	76.1	580
78	Slovakia	0	0.3	1.4	17,170
79	South Africa	0.057	17.4	35.7	7,610
80	Sri Lanka	0.021	7	29.1	2,920
81	Suriname	0.039	15.5	27.2	8,480
82	Swaziland	0.184	62.9	81	2,860
83	Syria	0.021	1.7	16.9	2,610
84	Tajikistan	0.068	21.5	50.9	860
85	Tanzania	0.367	67.9	87.9	570
86	Thailand	0.006	0.4	11.5	5,210
87	East Timor	0.36	37.4	72.8	3,670
88	Togo	0.284	38.7	69.3	500
89	Trinidad and Tobago	0.02	4.2	13.5	14,400
90	Tunisia	0.01	2.6	12.8	4,150
91	Turkey	0.028	2.7	9.1	10,830
92	Uganda	0.367	37.7	64.5	440
93	Ukraine	0.008	0	0.1	3,500
94	Uruguay	0.006	0	0.2	13,510
95	Vietnam	0.084	13.1	38.5	1,400
96	Yemen	0.283	17.5	46.6	1,110
97	Zambia	0.328	64.3	81.5	1,350

Methodology

Data from the 97 countries on Gross National Product (GNP), Multidimensional Poverty Index (MPI), and Poverty Incidence (PI) were used to obtain the fractal dimensions derived from the formula $\lambda = 1 + \frac{n}{\sum_{i=1}^{n} log\left(\frac{x_i}{\theta}\right)}; where\ 0.6 < \lambda < 3.4$ and the coefficient of variations CV where $CV = \frac{s}{\bar{\bar{x}}}$ of these variables.

Data were obtained from the net. Table 1 shows the data of the 97 countries with their corresponding Gross National Product (GNP), Multidimensional Poverty Index (MPI), and

Poverty Incidence (in percentage) with \$1.25/day and \$2.00/day income, respectively. At this point, if we observe a CV <1, then the amount of fluctuations around the mean is bounded, otherwise, unbounded.

From this observation, determine which of the variables (GNP and MPI) has likely influenced the PI of these countries considering the computed fractal dimensions is closest to the computed fractal dimension of the PI. Fractal Spectra of the three variables were then obtained by plotting their respective fractal dimensions against their scales and were analyzed. Correlations among variables were also computed based on the computed fractal dimensions.

Results and Discussions

Fractal Characteristics of the Variable

Table 2 shows the results obtained from the different measures of the variables MPI, PI and GNP. To determine whether the variables MPI, PI and GNP behave as fractals or not, coefficient of variations and fractal dimensions were considered. It is observed that the three variables have fractal characteristics. Further, it is observed that the fractal dimension of the MPI is closer to the fractal dimension of the PI which implies that this variable defines poverty incidence better. Looking at their respective CVs it is noted the amount of fluctuations around the mean of the MPI and GNP are unbounded and thus explains the large variations of data except for the PI, which is bounded. Thus, fractal characteristics are evident in these variables.

Looking at the given data, the irregularities or ruggedness of income distribution, in terms of GNP of the different countries was observed. There are more countries having low GNP than high GNP. These low-income countries likewise have greater poverty incidence compared to high-income countries.

Among those with low GNP, there are more having even lower income than higher income. The same is true for the high-income countries, there are more having lower income than higher income. Also, fluctuations of poverty incidence in poor countries have the same fluctuations of poverty incidence in not poor countries. Thus, this exhibits a self-similarity property.

Also, majority of the poor countries having high MPI have high poverty incidence, and some countries with low MPI have low poverty incidence. Huelgas (2011) and Vijayakumarf and Olga (2012) both revealed that employment and education were some factors that significantly affect the poverty incidence. These factors were also integrated in the MPI. Hence, MPI could explain the rugged and irregular behavior of poverty across the countries.

Table 2
Summary of Measures of the Variables

Variable	Fractal Dimension (λ)	Mean ($\bar{x}$)	Standard Deviation (sd)	Coefficient of Variation (CV)
MPI	1.278	0.180	0.181	1.015
PI	1.190	40.819	30.676	0.752
GNP	1.435	4,158.000	4,314.589	1.038

Fractal Spectra of the Variables

Fractal spectra of the three variables were then plotted by their respective fractal dimensions against their scales and were analyzed. These are on the poverty incidence, MPI and GNP.

Table 3
Lambda's at áth Percentile for PI

K	PI	α	log(1-α)	M (log(x / θ))	Scale (1 / M)	Fractal Dimension (λ)
1	0.1	0.010	-0.010	0	-	-
2	0.1	0.021	-0.021	0	-	-
3	0.1	0.031	-0.031	0	-	-
4	0.2	0.041	-0.042	0.693	1.443	1.061
5	0.2	0.052	-0.053	0.693	1.443	1.076
6	0.2	0.062	-0.064	0.693	1.443	1.092
7	0.2	0.072	-0.075	0.693	1.443	1.108
8	0.4	0.082	-0.086	1.386	0.721	1.062
9	0.7	0.093	-0.097	1.946	0.514	1.050
10	1	0.103	-0.109	2.303	0.434	1.047
11	1.4	0.113	-0.120	2.639	0.379	1.046
⋮	⋮	⋮	⋮	⋮	⋮	⋮
94	89.6	0.969	-3.476	6.798	0.147	1.511
95	90.5	0.979	-3.882	6.808	0.147	1.570
96	93.5	0.999	-4.575	6.841	0.147	1.669
97	94.8	1	-	6.854	0.146	-!

On Poverty Incidence

Table 3 presents the fractal dimension of the PI at the áth percentile while Figure 2 shows the spectrum of the PI. We notice that there are three (3) scales at different ranges. These scales will tell us where a certain country belongs.

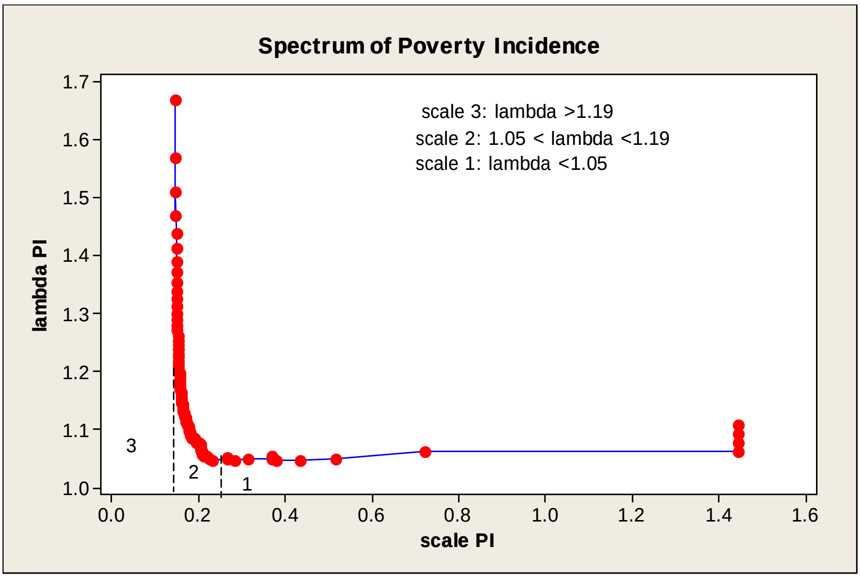

Figure 2. Spectrum of poverty incidence.

The lowest scale (scale 3) for which the fractal dimension is greater than 1.19 is composed of countries which have lesser poverty incidence but greater variability. These are mostly developed countries. The second scale for which fractal dimension is between 1.09 and 1.19 refers to the developing countries including the Philippines. The highest scale (scale 1), having the fractal dimension less than 1.09 is the poorest countries called the least developed countries. This is the group of countries that have been classified by the UN as "least developed" in terms of their low gross national income (GNI), their weak human assets and their high degree of economic vulnerability. Most of these are African countries. Appendix A shows the list of countries belonging to the three different scales under Poverty Incidence.

On Multidimensional Poverty Index

Fractal dimensions are again computed and graphed, as presented in Table 5 and Figure 3, respectively.

Table 5
Lambda's at áth Percentile for MPI

K	PI	α	log(1-α)	M (log(x / θ))	Scale (1 / M)	Fractal Dimension (λ)
1	0.000	0.010	-0.010	-	-	-
2	0.002	0.021	-0.021	0.000	-	-
3	0.003	0.031	-0.031	0.405	2.47	-
4	0.003	0.041	-0.042	0.405	2.47	1.08
5	0.003	0.052	-0.053	0.405	2.47	1.10
6	0.004	0.062	-0.064	0.693	1.44	1.13
7	0.005	0.072	-0.075	0.916	1.09	1.09
8	0.005	0.082	-0.086	0.916	1.09	1.08
9	0.006	0.093	-0.097	1.099	0.91	1.09
10	0.006	0.103	-0.109	1.099	0.91	1.09
11	0.006	0.113	-0.120	1.099	0.91	1.10
⋮						
94	0.536	0.969	-3.476	5.591	0.18	1.57
95	0.558	0.979	-3.882	5.631	0.18	1.62
96	0.562	0.990	-4.575	5.638	0.18	1.69
97	0.642	1.000	-	5.771	0.17	1.81

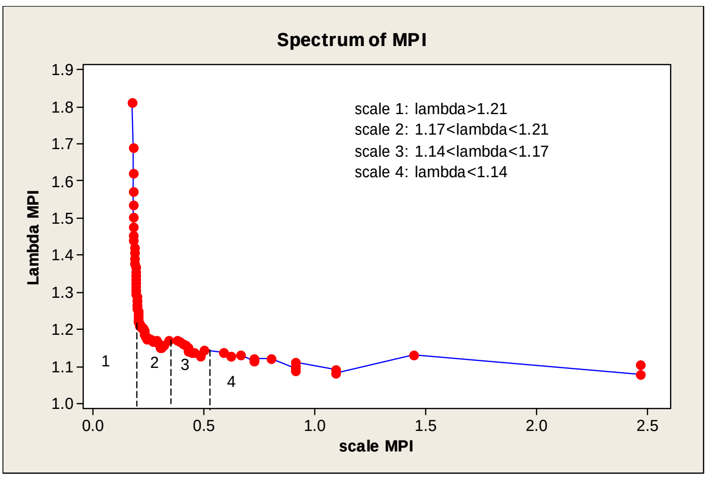

Figure 3. Fractal spectrum of Multidimensional Poverty Index.

As shown in Figure 3, four (4) different scales of MPI fractal spectrum are obtained. The lowest scale (scale 1) with fractal dimension greater than 1.17 is the least developed countries which include African countries. These are the most deprived countries in terms of basic services as indicated by their higher MPIs. The highest scale (scale 4) with fractal dimension lower than 1.14 is the countries with lower MPIs. Some of these are developed countries. Scale 3 is composed of group of developing countries (not so poor) while scale 2 is a group of developing countries including few least developed countries. The jump observed between scales 2 and 3 is due to countries which belong to the poorest countries but with not so high MPI. This implies that these countries may be poor in terms of its income but may have more access to some basic services than those of some developing countries. Appendix 2 shows the summary of countries at different scales of MPI.

On Gross National Product

Table 6 indicates the fractal dimension of the countries in terms of their respective GNP. As observed in Figure 4, the fractal spectrum

Table 6
Lambda's at áth Percentile for GNP

K	PI	α	log(1-α)	M=log(x/θ)	Scale=1/M	Fractal Dimension (λ)
1	240	0.010	-0.010	0.000	-	-
2	320	0.021	-0.021	0.288	3.48	1.07
3	370	0.031	-0.031	0.433	2.31	1.07
4	370	0.041	-0.042	0.433	2.31	1.10
5	410	0.052	-0.053	0.536	1.87	1.10
6	430	0.062	-0.064	0.583	1.71	1.11
7	440	0.072	-0.075	0.606	1.65	1.12
8	460	0.082	-0.086	0.651	1.54	1.13
9	490	0.093	-0.097	0.714	1.40	1.14
10	500	0.103	-0.109	0.734	1.36	1.15
⋮	⋮	⋮	⋮	⋮	⋮	⋮
94	14400	0.969	-3.476	4.094	0.24	1.849
95	15830	0.979	-3.882	4.189	0.24	1.927
96	17170	0.990	-4.575	4.270	0.23	2.071
97	18130	1.000	-	4.325	0.23	-

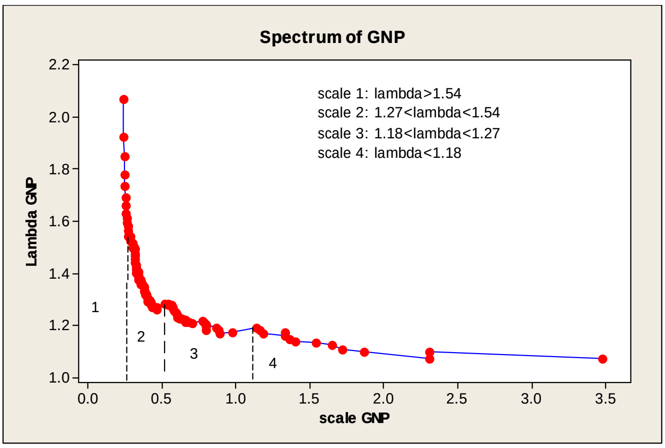

Figure 4. Fractal spectrum of Gross National Product.

of GNP has four (4) scales. Countries with very low gross national income are countries belonging to scale 4. Majority are African countries. The Philippines belongs to Scale 2, which is a group of average – income or developing countries. Developed countries or high-income countries belong to scale 1. Appendix 3 gives the list of these countries.

Figure 5 shows clearly the variable that defines poverty incidence better. The spectrum of MPI fits more likely to the spectrum of PI than the spectrum of GNP. This indicates that the ruggedness of poverty incidence across the countries is due to the ruggedness of the MPI. Deprivation of the countries to basic services such as health, education and standard of living affects the quality of living.

Furthermore, the percentage of poor countries under PI spectrum is compared to the poor countries in MPI and GNP spectrum. Appendix 4 gives the list of poor countries in each variable. Forty-four percent (44%) of the 79 countries are poor in terms of MPI, 35% are poor in terms of poverty incidence and 17% are poor in terms of GNP. Seventeen percent (17%) of these have low-income and were deprived of basic services.

Moreover, 14% of the countries have high poverty incidence at the same time have low income. Thirty four percent (34%) have high poverty incidence at the same time most deprived in terms of basic services. All poor countries in terms of GNP are also poor in terms of MPI, however, 27% are poor in terms of MPI but not in GNP. This implies that wealth alone cannot explain the rugged behavior of poverty. The ruggedness of MPI could be closely related to the ruggedness in poverty incidence.

Correlation Analysis

Correlations among variables are also computed based on the computed fractal dimensions. This is to determine which of the two variables, the GNP or the MPI, explains poverty more precisely. Results show the regression equation between the PI and MPI, and PI and GNP, respectively as follows:

$$\lambda_{PI} = 0.425 + 0.603\ \lambda_{MPI}$$

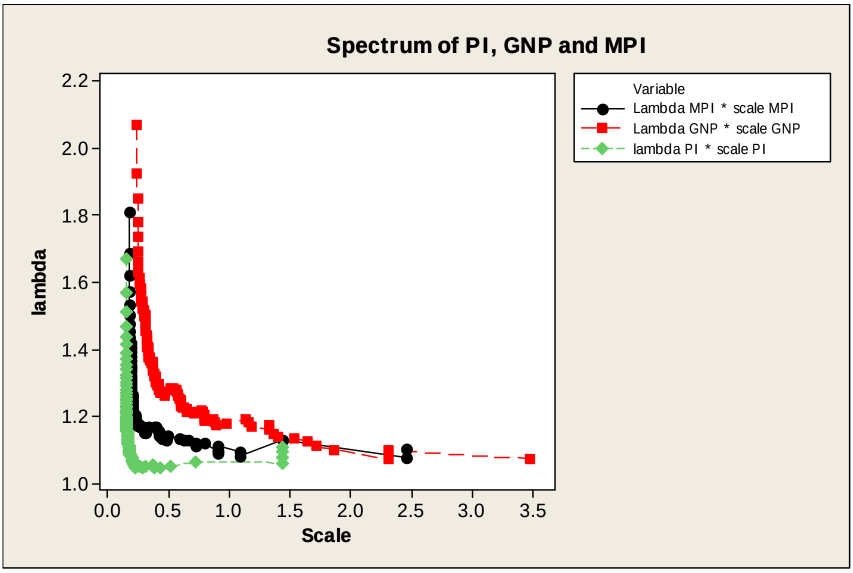

Figure 5. Super-imposed spectra of PI, MPI and GNP.

and

$$\lambda_{PI} = 1.210 - 0.301\ \lambda_{GNP}$$

Tables 7 and 8 further show the significant difference between the fractional dimensions of PI and MPI, versus PI GNP. Results confirmed that MPI is closely related to poverty incidence, but not GNP. In fact, 42.8% of the variation in poverty incidence can be explained by the variation in MPI. However, only 0.2% can be explained by GNP. Hence, MPI explains poverty incidence more precisely. This is consistent with our analysis using spectrum and fractal dimensions.

Table 7
Test of Significant Difference between PI and MPI

Predictors	Coef	SE Coef	T	P
Constant	0.42542	0.09216	4.62	0.000
λ_{MPI}	0.60311	0.07435	8.11	**0.000**

S = 0.0970329 **R-Sq = 42.8%** R-Sq(adj) = 42.1%

Table 8
Test of Significant Difference between PI and GNP

Predictors	Coef	SE Coef	T	P
Constant	1.29502	0.09630	12.51	0.000
λ_{GNP}	-0.03065	00.07100	-0.43	**0.667**

S = 0.126977 **R-Sq = 0.2%** R-Sq(adj) = 0.0%

Conclusion

Based on the results and discussions, poverty incidence of the 97 countries can be explained better by MPI. MPI looks at the deprivation of basic services at the household level while GNP focuses on the income of every citizen of a country. In other words, GNP is directly proportional to the quality of living while MPI is inversely proportional to the standard of living. Thus, the policymakers should focus on programs and projects for poverty alleviation particularly on the basic services.

References

Alkire, S., Roche, S., Santos, M.E., & Seth S. (2011). *Multidimensional poverty index: 2011 data.* Oxford Poverty and Human Development Initiative. Retrieved October 5, 2012, from http://ophi.qeh.ox.ac.uk

Chattopadhyay, A.K., & Mallick, S.K. (2006). *Income distribution dependence of poverty measure: A theoretical analysis.* Retrieved November 21, 2012, from http://dx.doi.org/10.1016/j.physa.2006.10.103

Coramaldi, M., & Zoli, M. (2011). *Income distribution dependence of poverty measure: A theoretical analysis.* Springer Sciences Business Media B.V.

Huelgas, A., (2013). *Classical and spatial regression techniques in estimating city and municipal poverty incidence in the Philippines.* A paper presented during the 12th National Convention on Statistics (NCS). EDSA Shangri-La Hotel, Mandaluyong City, Philippines.

Oxford Poverty and Human Development Initiative. (2011). *Multidimensional poverty index 2011.*

Shah, A., (2013). Poverty Facts and Stats. Global Issues.Retrieved July 26, 2013, from www.globalissues.org/article/26

Tzavidis, N., & Salvati, N. (2007). *M-quantile models with application to poverty mapping.* Center for Census and Survey Research.

Vijayakumarf, B., & Olga, S. (2012, March). Poverty incidence and its determinants in the estate sector of Sri Lanka. *Journal of Competitiveness,* 4 (1), 44-45.

Appendix A

Summary of Countries at Different Scales of Poverty Incidence (PI)

Countries	PI	Fractal Dimension, (λ)	Scale Category
Slovakia	1.4	1.045609	3
Azerbaijan	7.8	1.047114	3
Jordan	3.5	1.04725	3
Latvia	1	1.047252	3
Albania	4.3	1.047927	3
Estonia	1.5	1.048766	3
Mexico	8.6	1.048941	3
Argentina	2.4	1.049046	3
Serbia	0.7	1.05004	2
Turkey	9.1	1.051189	2
Macedonia	4.3	1.051229	2
Brazil	9.9	1.053095	2
Kazakhstan	1.5	1.053136	2
Thailand	11.5	1.05421	2
Armenia	12.4	1.056147	2
Moldova	12.5	1.058872	2
Montenegro	0.2	1.060754	2
Tunisia	12.8	1.061427	2
Hungary	0.4	1.06209	2
Paraguay	13.2	1.063904	2
Trinidad and Tobago	13.5	1.066503	2
Ecuador	13.6	1.069332	2
Dominican Republic	13.6	1.072304	2
Morocco	14	1.074878	2
Uruguay	0.2	1.076351	2
Peru	14.7	1.077159	2
Syria	16.9	1.078037	2
Guyana	18	1.080075	2
Egypt	18.5	1.082672	2
Guatemala	24.3	1.084441	2
Gabon	19.6	1.084799	2
Belize	24.5	1.087319	2
Bolivia	25.1	1.089979	2
Czech Republic	0.2	1.092118	2
Iraq	25.3	1.092939	2
Suriname	27.2	1.094841	2
Colombia	27.9	1.097556	2
Sri Lanka	29.1	1.100008	2
Kyrgyzstan	29.4	1.103056	2
Cameroon	30.4	1.105723	2
Bosnia and Herzegovina	0.2	1.10806	2
Nicaragua	31.9	1.108143	2
Georgia	32.2	1.111331	2
Honduras	35.4	1.112908	2
South Africa	35.7	1.116183	2
China	36.3	1.119352	2
Vietnam	38.5	1.121709	2
Kenya	39.9	1.124574	2
Djibouti	41.2	1.127561	2
Mauritania	44.1	1.129827	2
Philippines	45	1.133161	2
Cote d'Ivoire	46.3	1.136376	2
Yemen	46.6	1.140155	2
Bhutan	49.5	1.142771	2
Indonesia	50.6	1.146333	2
Tajikistan	50.9	1.150362	2
Ghana	53.6	1.153369	2
São Tomé and Príncipe	55.9	1.156682	2
Cambodia	56.5	1.160863	2
Gambia	56.7	1.165346	2
Senegal	60.4	1.168375	2
Pakistan	61	1.172914	2
Namibia	62.2	1.177325	2
Lesotho	62.3	1.182377	2
Uganda	64.5	1.186639	2
Comoros	65	1.191834	1
Laos	66	1.196985	1
Togo	69.3	1.201285	1
Guinea	69.6	1.207145	1
Angola	70.2	1.213102	1
Haiti	72.2	1.218659	1
East Timor	72.8	1.225129	1
Congo, Republic of the	74.4	1.231425	1
Benin	75.3	1.23837	1
India	75.6	1.245966	1
Niger	75.9	1.253971	1
Sierra Leone	76.1	1.262486	1
Mali	77.1	1.27109	1
Ethiopia	77.6	1.280526	1
Congo, Democratic	79.6	1.289786	1
Central African Republic	80.1	1.300599	1
Swaziland	81	1.312049	1
Burkina Faso	81.2	1.324922	1
Bangladesh	81.3	1.339086	1
Zambia	81.5	1.35468	1
Mozambique	81.8	1.372047	1
Chad	83.3	1.390898	1
Nigeria	83.9	1.413379	1
Tanzania	87.9	1.437434	1
Madagascar	89.6	1.469027	1
Rwanda	89.6	1.511346	1
Malawi	90.5	1.570153	1
Burundi	93.5	1.668764	1
Russia	0.1		
Ukraine	0.1		
Croatia	0.1		
Liberia	94.8		

Appendix 2

Summary of Countries at Different Scales of Multidimensional Poverty Index (MPI)

Countries	MPI	Fractal Dimension, (λ)	Scale Category
Georgia	0.003	1.077	4
Russia	0.005	1.082	4
Montenegro	0.006	1.089	4
Albania	0.005	1.092	4
Latvia	0.006	1.094	4
Thailand	0.006	1.099	4
Serbia	0.003	1.104	4
Uruguay	0.006	1.11	4
Macedonia	0.008	1.112	4
Jordan	0.008	1.1151	4
Moldova	0.007	1.12	4
Ukraine	0.008	1.121	4
Tunisia	0.01	1.128	4
Croatia	0.016	1.128	4
Czech Republic	0.01	1.128	4
Ecuador	0.009	1.13	4
Hungary	0.016	1.13	4
Armenia	0.004	1.131	4
Brazil	0.011	1.135	4
Argentina	0.011	1.135	4
Kyrgyzstan	0.019	1.136	4
Dominican Republic	0.018	1.137	4
Trinidad and Tobago	0.02	1.139	4
Azerbaijan	0.021	1.142	3
Mexico	0.015	1.143	3
Sri Lanka	0.021	1.145	3
Syria	0.021	1.151	3
Guyana	0.053	1.151	3
China	0.056	1.152	3
South Africa	0.057	1.154	3
Morocco	0.048	1.156	3
Colombia	0.022	1.157	3
Iraq	0.059	1.159	3
Belize	0.024	1.161	3
Egypt	0.024	1.161	3
Paraguay	0.064	1.162	3
Philippines	0.064	1.164	3
Peru	0.086	1.167	3
Estonia	0.026	1.167	3
Turkey	0.028	1.168	3
Tajikistan	0.068	1.169	3
Suriname	0.039	1.17	3
Bolivia	0.089	1.171	2
Vietnam	0.084	1.171	2
Guatemala	0.127	1.172	2
Nicaragua	0.128	1.174	2
Indonesia	0.095	1.175	2
Bhutan	0.119	1.177	2
Djibouti	0.139	1.179392	2

Countries	MPI	Fractal Dimension, (λ)	Scale Category
Ghana	0.144	1.181087	2
São Tomé and Príncipe	0.154	1.184845	2
Lesotho	0.156	1.187281	2
Honduras	0.159	1.192127	2
Gabon	0.161	1.196798	2
Namibia	0.187	1.201502	2
Swaziland	0.184	1.201864	2
Congo, Republic of the	0.208	1.206508	2
Kenya	0.229	1.207518	2
Cambodia	0.251	1.209087	2
Pakistan	0.264	1.210948	1
Laos	0.267	1.214703	1
India	0.283	1.220307	1
Yemen	0.283	1.223931	1
Togo	0.284	1.230342	1
Cameroon	0.287	1.236795	1
Bangladesh	0.292	1.24312	1
Haiti	0.299	1.249319	1
Nigeria	0.31	1.255402	1
Gambia	0.324	1.261056	1
Zambia	0.328	1.266498	1
Chad	0.344	1.273862	1
Mauritania	0.352	1.279596	1
Cote d'Ivoire	0.353	1.28695	1
Madagascar	0.357	1.295785	1
East Timor	0.36	1.304552	1
Tanzania	0.367	1.313939	1
Uganda	0.367	1.323152	1
Malawi	0.381	1.334119	1
Senegal	0.384	1.343284	1
Congo, Democratic	0.393	1.355048	1
Comoros	0.408	1.366555	1
Benin	0.412	1.377908	1
Rwanda	0.426	1.39224	1
Sierra Leone	0.439	1.406024	1
Angola	0.452	1.421439	1
Liberia	0.485	1.438607	1
Guinea	0.506	1.454429	1
Central African Republic	0.512	1.47508	1
Mozambique	0.512	1.501869	1
Burundi	0.53	1.534748	1
Burkina Faso	0.536	1.571428	1
Mali	0.558	1.621733	1
Ethiopia	0.562	1.689295	1
Niger	0.642	1.811356	1
Slovakia	0		
Kazakhstan	0.002		
Bosnia and Herzegovina	0.003		

Appendix 3

Summary of Countries at Different Scales of Gross National Product (GNP)

Countries	GNP	Lambda	Scale
Malawi	320	1.072421	4
Niger	370	1.072578	4
Liberia	370	1.097286	4
Ethiopia	410	1.098825	4
Madagascar	430	1.109495	4
Uganda	440	1.123572	4
Guinea		1.132303	4
Central African Republic	490	1.136423	4
Togo	500	1.148239	4
Gambia	510	1.159682	4
Rwanda	560	1.169827	4
Chad	740	1.171121	4
Mozambique	510	1.175199	4
Burkina Faso	670	1.175584	4
Benin	750	1.180145	3
Tanzania	570	1.180197	3
Kenya	840	1.184317	3
Haiti	760	1.189126	3
Sierra Leone	580	1.190383	3
Comoros	840	1.194752	3
Bangladesh	840	1.205324	3
Kyrgyzstan	990	1.210325	3
Tajikistan	860	1.212056	3
Senegal	1040	1.212796	3
Mauritania	1110	1.213007	3
Cambodia	880	1.218776	3
Yemen	1110	1.222403	3
Cameroon	1170	1.224227	3
Cote d'Ivoire	1220	1.227568	3
Pakistan	1260	1.232209	3
Laos	1260	1.241417	3
Djibouti	1280	1.248407	3
São Tomé and Príncipe	1320	1.253161	3
Zambia	1350	1.259131	3
Moldova	2070	1.263325	3
Lesotho	1380	1.265171	3
Bhutan	2420	1.269796	3
Bolivia	2220	1.271695	2
Honduras	2070	1.271841	2
Vietnam	1400	1.27238	2
Philippines	2470	1.275759	2
India	1530	1.277625	2
Nigeria	1430	1.278561	2
Congo, Republic of the	2550	1.280419	2
Nicaragua	1650	1.284953	2
Ghana	1550	1.285015	2
Congo, Democratic	2550	1.288967	2
Swaziland	2860	1.292406	2
Syria	2610	1.294791	2
Sri Lanka	2920	1.298583	2
Morocco	2940	1.306542	2
Egypt	3000	1.312987	2
Guatemala	3120	1.317164	2
Georgia	3280	1.320097	2
Paraguay	3290	1.32893	2
Guyana	3410	1.333794	2
Indonesia	3420	1.342955	2
Ukraine	3500	1.349689	2
East Timor	3670	1.353386	2
Albania	4090	1.35948	2
Armenia	3720	1.361638	2
Tunisia	4150	1.367813	2
Angola	4580	1.376075	2
Belize	4180	1.377334	2
Bosnia and Herzegovina	4650	1.384862	2
Macedonia	4690	1.394784	2

Appendix 4

List of Countries Belonging to Poor Countries in Three Variables

PI	GNP	MPI
Angola	Burkina Faso	Angola
Bangladesh	Central African Republic	Bangladesh
Benin	Chad	Benin
Burkina Faso	Ethiopia	Burkina Faso
Burundi	Gambia	Burundi
Central African Republic	Guinea	Cameroon
Chad	Liberia	Central African Republic
Comoros	Madagascar	Chad
Congo, Democratic	Malawi	Comoros
Congo, Republic of the	Mozambique	Congo, Democratic
East Timor	Niger	Cote d'Ivoire
Ethiopia	Rwanda	East Timor
Guinea	Togo	Ethiopia
Haiti	Uganda	Gambia
India		Guinea
Laos		Haiti
Madagascar		India
Malawi		Laos
Mali		Liberia
Mozambique		Madagascar
Niger		Malawi
Nigeria		Mali
Rwanda		Mauritania
Sierra Leone		Mozambique
Swaziland		Niger
Tanzania		Nigeria
Togo		Pakistan
Zambia		Rwanda
		Senegal
		Sierra Leone
		Tanzania
		Togo
		Uganda
		Yemen
		Zambia