
Fractal Analysis of the Academic Performance of College Students in Their Different Subjects

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Abstract

This paper investigated the fractal characteristics of the academic performance of college students. Final grades of the students in Mathematics, English, Science and Filipino were used to measure their academic performance. Fractal dimensions were obtained, and the self-similarity of the academic performance of students across colleges was investigated using fractal spectrum. Results revealed that the academic performance of students is fractal in nature but bounded. Self-similarity is also found in students' academic performance in the four subjects mentioned across colleges. With this finding, implications to educators to understand and improve academic performance of students can be addressed.

Keywords: Fractal analysis, students' performance, education, spectrum, ruggedness

Introduction

Humans resemble humans. Each of us shares a long list of intrinsic similarities to all other humans. These similarities extend across the sexes, races, and cultures and include many details of anatomy, behavior, and mental processes. At our core, we all share common aspects of being human like creativity, learning, and brain function. As humans, we have much more in common than the great diversity of our species might seem to indicate (Brown, 1991).

Perhaps this commonality is not surprising. The laws of logic, mathematics, physics, chemistry, and economics hold uniformly true throughout the known universe. Humans everywhere share a common and complex anatomy, physiology, genome, and brain structure. Every person alive today is descended from the Sand Bushmen who left their African village only 60,000 years ago to populate the far reaches of the globe. Perhaps our common traits result from the strategies our selfish genes used to survive the clever and

relentless competition they face on our remarkable planet (Wells, 2004).

Being aware of this core of commonality can have a positive impact on what goes on in the places where we work. It can help us find new and better ways to bring out the best in each other in terms of creativity, learning, and getting things done in an increasingly distracting and hurried world.

Success of students is measured by their academic performance, or how well a student meets standards set out by the local government and the institution itself (Bell, 2013). Efforts on identifying, evaluating, tracking and encouraging the progress of students in school should be taken seriously since it affects both the performance of the students, the teachers and the institution.

The deteriorating performance of Filipino students in the national and international mathematics tests in the last decade has become a major challenge to Philippine

education. The Department of Education attributed this problem to students' poor reading comprehension (Rimando, 2006). Previous studies showed varied findings on the association between variables in reading and mathematics (Imam, 2010).

Imam, Abas-Mastura, and Jamil (2013) used correlation research design to determine the relationship between reading comprehension and mathematics. Results showed that reading comprehension skills were insignificantly correlated to private school students' mathematics performance. But the case is different in public schools where three skills namely understanding vocabulary in context, getting the main idea, and making inference surfaced to have connection with mathematics. The overall students' reading comprehension skills were not significantly correlated to mathematics performance. Hence, poor mathematics performance could be explained by other factors not related to reading comprehension skills.

On the field of education, there are subjects that are more difficult compared to other subjects and it is interesting to know whether this commonality is shared among all students or of students with the same course. It is also important to know whether the performances of the students in one subject have relationship with the other subjects the student is currently taking.

Recent efforts in analyzing data have been connected to the world of fractal analysis. Many education researchers have been using the usual method in analyzing data through the mean, standard deviation, one-way and/or two-way ANOVA, chi-square and the like. This kind of analysis approximates only the inclusion of the scores that are closer to each other and neglects the extreme value scores. There are no researches yet conducted on fractals on this undertaking, when in fact, there are a lot of data interpreted as such but are not distributed

normally. Thus, the researchers thought of conducting this study and used fractal analysis in analyzing the data.

A structure is called fractal if it shows self-similarity in a somewhat technical sense, on all scales and if it has a non-integer dimension. The object need not exhibit exactly the same structure at all scales, but the same "type" of structures must appear on all scales. The universe is a fractal made of fractals. Any kind of pattern can be a fractal, including intervals of time, the positions of things within an area, behavior, the evolution of something, and physical objects. In other words, any kind of system can be a fractal. Nature is made up of fractals. Earthquakes, lightning, pinecones, trees, ferns, the positions of trees in a forest, the distribution of acid in a rainfall, and coastlines are all examples of fractals (as is everything) in nature. The rhythm of heartbeats, the development of AIDS, the circulatory system, and bronchial tubes inside lungs are examples of fractals in the human body. The chaos in the ebb and flow of economics and crowds make them fractal too. Because the universe is made of them, fractal science can be applied to everything for an estimated prediction of the future course of things, as well as to better understand the universe itself.

Fractals have been recently adjoined with the creation of micro-chips and electronic technologies, more specifically on the creation of signal capturing devices, like antennas, as well as on the internet traffics (Hohlfeld & Cohen, 1999). Fractal models have also been connected to biology, physics, environment, governance and many other fields of interests. This paper however intends to link fractals and education; it aims to investigate the fractal nature of the academic performance of college students.

Fractal analysis is an analysis that assesses the fractal characteristics of data. It consists of several methods to assign a fractal dimension

and other fractal characteristics to a dataset applying nontraditional mathematics to patterns that defy understanding with traditional Euclidean concepts. It is an analysis using fractal statistics which is concerned with data irregularities repeated at different scales and generalizing the concept of variances. It measures complexity using the fractal dimension.

Conceptual Framework

Fractal statistics is concerned with data irregularities repeated at different scales generalizing the concept of variances. The data are fractals when the variances are too large such that the coefficient of variation (CV) is greater than 1, and if there are lower values than higher values (Padua, 2014).

Self-Similarity at Various Scales

A function $f(x)$ is self-similar (or homogeneous of degree k) if

$$f(bx) = b^k f(x), k \text{ is any real number. (1)}$$

The only self-similar function in one variable is the monomial function

$$f(x) = ax^k, k \text{ is any real number. (2)}$$

From the class of self-similar functions, a subclass that gives larger weights to the lower values is obtained, that is, the function

$$f(x) = ax^{-\lambda} \text{ where } \lambda > 0, x > 0. (3)$$

Given $f(x)$ in (3), we want to convert this into a probability distribution such that

$$\int_{-\infty}^{\infty} f(x) dx = 1. (4)$$

Using (4), $\int_{\theta}^{\infty} ax^{-\lambda} dx = 1$, solve for a . Then

$$a = \frac{1}{\int_{\theta}^{\infty} ax^{-\lambda} dx}$$

$$a = \frac{\lambda-1}{\theta^{\lambda-1}}, \lambda > 1. (5)$$

Quantitative Model for Fractal Statistics

A random variable x is said to behave in a fractal distribution if it obeys a power-law:

$$f(x) = \frac{\lambda-1}{\theta^{\lambda-1}} \left(\frac{x}{\theta}\right)^{-\lambda}, \lambda > 1 \text{ and}$$

$$\theta = \text{minimum}\{x\} (6)$$

where λ is called the fractal dimension of the distribution.

The model in (6) has two parameters, θ and λ , which are both unknown. To estimate the value of θ , we get the minimum of the data, that is,

$$f(x) = \frac{\lambda-1}{\theta^{\lambda-1}} \left(\frac{x}{\theta}\right)^{-\lambda}, \lambda > 1 \text{ and } \theta = \text{minimum}\{x\}. (7)$$

To estimate the value of λ , we use the Maximum Likelihood Estimator (MLE).

Maximum Likelihood Estimator of λ

Given the observations, from the fractal distribution $f(x)$ in (6), the likelihood function L is obtained.

$$L = \prod_{i=1}^n f(x_i) = \left(\frac{\lambda-1}{\theta}\right)^n \prod_{i=1}^n \left(\frac{x_i}{\theta}\right)^{-\lambda} (8)$$

Taking the logarithm of L ,

$$\log L = n \log \left(\frac{\lambda-1}{\theta}\right) - \lambda \sum_{i=1}^n \log \left(\frac{x_i}{\theta}\right). (9)$$

The MLE of θ is the value that maximizes the (8) or (9) by taking its derivative with respect to θ and equate to zero:

$$\frac{\partial \log L}{\partial \lambda} = 0$$

$$\frac{\partial \log L}{\partial \lambda} = n \cdot \frac{1}{\lambda - 1} - \sum \log \left(\frac{x_i}{\theta} \right) = 0$$

Solving for the estimate of λ ,

$$\hat{\lambda} = 1 + \frac{n}{\sum \log \left(\frac{x_i}{\theta} \right)} \tag{10}$$

Fitting a Fractal Distribution to Data

Given the observations, $x_1, x_2, x_3, \dots, x_n$ from the fractal distribution $f(x)$, arrange them such that $x_1 < x_2 < \dots < x_k < \dots < x_n$. Weights denoted by α_k are assigned to $x_{(k)}$,

$$\alpha_k = \frac{k}{n}, \quad k = 1, 2, 3, \dots, n \tag{11}$$

The percentile of the distribution obeys the rule

$$\int_{\theta}^{x_{\alpha}} f(x) dx = \alpha \tag{12}$$

$$\int_{\theta}^{x_{\alpha}} \frac{\lambda - 1}{\theta^{\lambda-1}} \left(\frac{x}{\theta} \right)^{-\lambda} dx = \alpha \tag{13}$$

Solving for λ_{α} , (13) gives

$$\lambda_{\alpha} = 1 - \frac{\log_e(1-\alpha)}{\log_e \left(\frac{x_{\alpha}}{\theta} \right)} \tag{14}$$

Letting $S = \frac{1}{\log \left(\frac{x_{\alpha}}{\theta} \right)}$, then

$$\lambda_{\alpha} = 1 - \ln(1 - \alpha)S \tag{15}$$

S is called the scale of fractal spectrum. The fractal spectrum is the visual and graphical representation of the changes in fractal dimensions as function of scales S .

If the performance of college students in the four subjects is fractal in nature, then educators can focus on subjects that are more difficult than others to reduce the differences among subjects (smoothen the fractal dimension).

Objectives

The paper attempted to explain the academic performance using fractal analysis. More specifically, this study tried to investigate the following research questions:

1. Is the academic performance of college students in Math, Science, Filipino and English fractal in nature?
2. Do all college students share commonality in terms of level of difficulty in their subject in Math, Science, Filipino and English?
3. Which of the following subjects in Math, Science, Filipino and English could more likely explain the performance of the students in the other subjects?

Methodology

The final grades of the first year college students in their four subjects offered during the first semester of the school year 2013-2014 were used to obtain the fractional dimensions (using

$$\lambda = 1 + \frac{n}{\sum_{i=1}^n \log \left(\frac{x_i}{\theta} \right)}; \text{ where } 0.6 < \lambda < 3.4$$

and the coefficient of variations (using

$$CV = \frac{s}{\bar{x}}) \text{ of these variables.}$$

The selection of the participants of this study is based on the following two steps: (1) four sections each from the four out of the five colleges were randomly chosen, such as, College of Arts and Sciences (CAS), College of Business, Accountancy, Hospitality and Public Governance (CBAHPG), College of Community Education and Industrial Technology (CCEIT), and College of Teacher Education (CTE), while the College of Nursing (CON) was not considered because it has only one first year section; and (2) twenty (20) students from each section were randomly chosen while considering that each student has taken all the four subjects: Math, English, Filipino and Science. The data gathered were the final grades of the students in all the four subjects mentioned above.

If the coefficient of variations of the variables is less than 1 ($CV < 1$) then the amount of fluctuations around the mean is

bounded, otherwise, unbounded. Fractal spectrum of the variables were obtained and analyzed by plotting their respective fractal dimensions (λ) against their scales (S).

Results and Discussion

Fractal Characteristics of the Variable

Table 1 shows the fractal characteristics of the academic performance of the students in all the four subjects in English, Math, Filipino and Science. To determine whether the academic performance of the student behaves as fractals or not, fractal dimensions and coefficient of variations were considered. From the table, it is observed that all the variables have fractal characteristics since their Fractal Dimensions (λ) lie between 0.6 to 3.4. Looking at their respective CVs it is noted that the

Table 1

Fractal Characteristics of the Academic Performance of the Students

College of Arts and Sciences (CAS)				
	English	Math	Filipino	Science
Fractal Dimension (λ)	1.4784	1.8651	1.6615	1.6835
Coefficient of Variations (C.V.)	0.4975	0.3746	0.2921	0.5814
College of Business, Accountancy, Hospitality and Public Governance (CBAHPG)				
	English	Math	Filipino	Science
Fractal Dimension (λ)	2.0182	1.8076	1.8161	1.7324
Coefficient of Variations (C.V.)	0.5192	0.4944	0.4804	0.3416
College of Community Education and Industrial Technology (CCEIT)				
	English	Math	Filipino	Science
Fractal Dimension (λ)	1.9017	1.8628	1.4720	1.8233
Coefficient of Variations (C.V.)	0.5760	0.5727	0.3388	0.4398
College of Teacher Education (CTE)				
	English	Math	Filipino	Science
Fractal Dimension (λ)	1.6909	2.2112	1.9616	2.2552
Coefficient of Variations (C.V.)	0.4009	0.3202	0.1999	0.2917

amount of fluctuations around the mean of all the four subjects are bounded since all the CVs are less than 1 and thus explains the very slim variations of data.

Fractal Spectrum of the Variable

The spectrum of CAS students' performance in English, Math, Filipino and Science is divided into three scales. The first scale consists of high performing students, the second consists of average performing students in that subject and the last consists of low performing students. In English, their fractal dimension of the low performing students is less than 1.10, less than 1.31 in Math, less than 1.09 in Filipino and less than 1.32 in Science. The first scale has greater variability in scores

compared to the other two scales. We are interested only in the first scale which is the lowest scale where the high-performing students belong. The high-performing students in Math have fractal dimensions greater than 2.16, greater than 1.59 in Science, greater than 1.50 in Filipino, and greater than 1.45 in English. This further implies that the high performing students in CAS who belong to scale 1 found Math to be most difficult among the four subjects since there are lesser high performing students who belong to this scale, followed by Science, then Filipino and least difficult in English. The superimposed graph is shown below to compare the variability of scores of CAS students in the four subjects.

As seen in Figure 2, the spectrum of English, Science and Filipino of the high

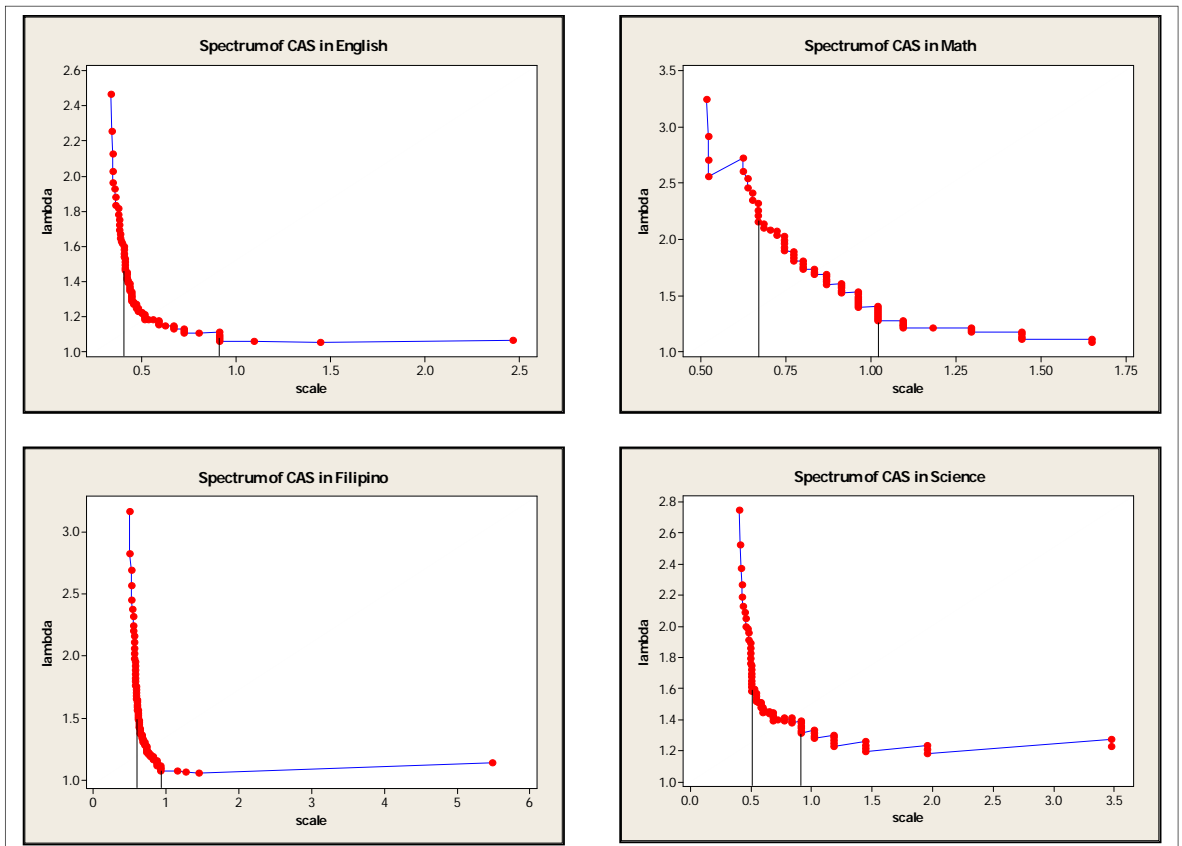


Figure 1. Spectrum of CAS in four subjects.

Table 2
Scales of the Spectrum of CAS in Four Subjects

	English	Math	Filipino	Science
Scale 1	$\lambda_\alpha > 1.45$	$\lambda_\alpha > 2.16$	$\lambda_\alpha > 1.50$	$\lambda_\alpha > 1.59$
Scale 2	$1.10 < \lambda_\alpha < 1.45$	$1.31 < \lambda_\alpha < 2.16$	$1.09 < \lambda_\alpha < 1.50$	$1.32 < \lambda_\alpha < 1.59$
Scale 3	$\lambda_\alpha < 1.10$	$\lambda_\alpha < 1.31$	$\lambda_\alpha < 1.09$	$\lambda_\alpha < 1.32$

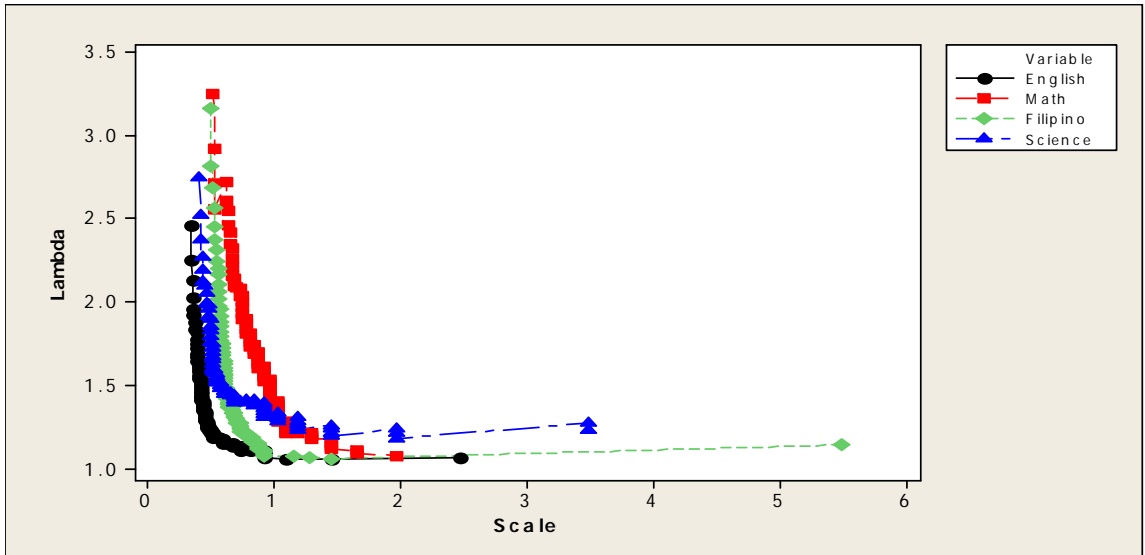


Figure 2. Spectrum of CAS in the four subjects.

performing students in CAS fits more likely with each other than in Math. This indicates that the ruggedness of the performance of CAS students in scale 1 in either one of these three subjects: English, Science and Filipino, could be explained by the ruggedness of the performance of the high performing students in the other two subjects. Notice also that the spectrum of the low performing students in English, Math and Filipino fits more likely with each other than in Science. This suggests that the ruggedness of the performance of CAS students in scale 3 in either one of these three subjects: English, Math and Science, could be explained by the ruggedness of the other two in scale 3 than in their science subject. In this

scale however, the low performing students perform better in their subject in science compared to the other three subjects.

The spectrum of the performance of the students in CBAPHG in English, Math, Filipino and Science also has three scales. In English, the students who belong to the third scale has fractal dimension less than 1.35, less than 1.34 in Math, less than 1.28 in Filipino and less than 1.20 in Science. The high-performing students who belong to the first scale have fractal dimensions greater than 1.86 in English, greater than 1.79 in Math, greater than 1.55 in Filipino, and greater than 1.46 in Science. This implies that the high performing students in CBAPHG performed better in their Science subject

compared to the other three since there are more high performing students who belong to this scale in this subject. Science is followed by Filipino, then Math and then English. The superimposed graph is shown below to compare the variability of scores of CBAPHG students in four subjects.

As seen in Figure 4, the spectrum of Math, Science and Filipino fits more likely with each other than in English. This indicates that the ruggedness of the performance of CBAPHG

students in either one of these three subjects: Math, Science and Filipino, could be explained by the ruggedness of the other two.

The spectrum of the performance of the students in CCEIT in the four subjects again has three scales as shown in Figure 5. The students in the first scale have the following fractal dimensions: greater than 1.78 in English, greater than 1.75 in Math, greater than 1.73 in Filipino and greater than 1.60 in Science. These data imply that the high performing students in

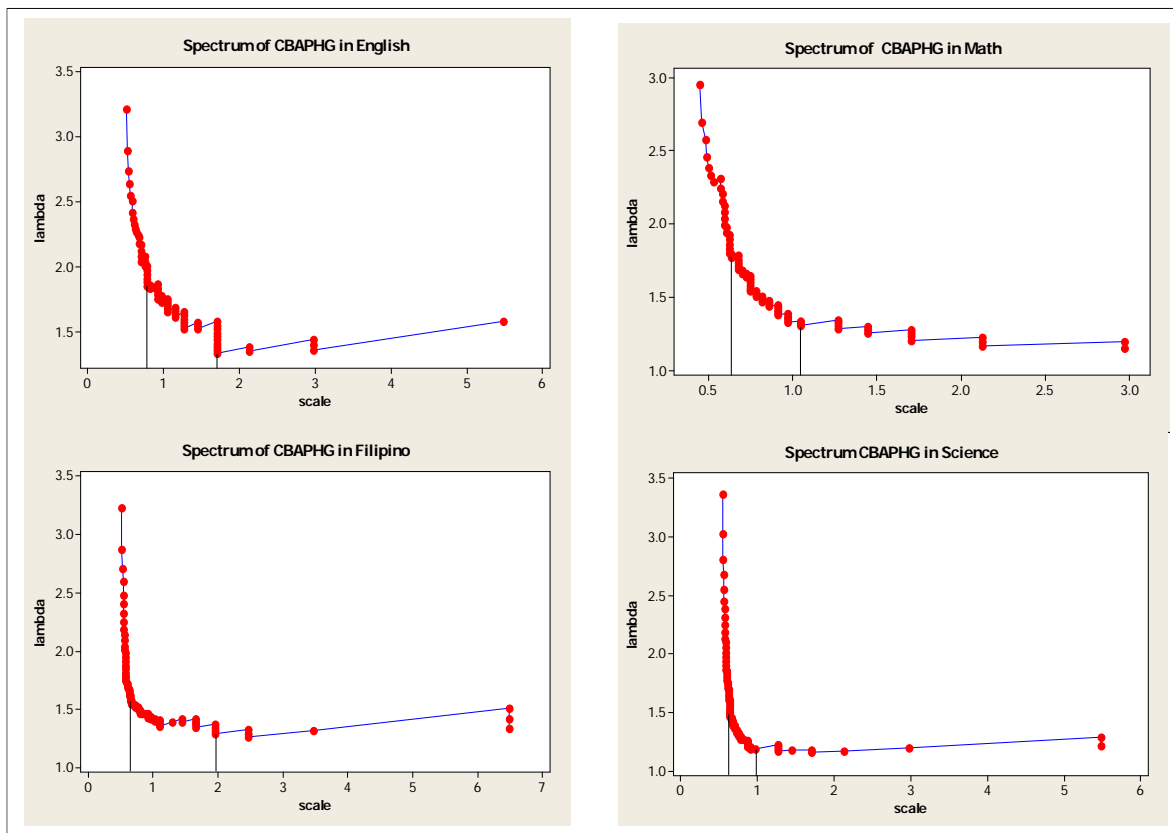


Figure 3. Spectrum of CBAPHG in four subjects.

Table 3

Scales of the Spectrum of CBAPHG in Four Subjects

	English	Math	Filipino	Science
Scale 1	$\lambda_\alpha > 1.86$	$\lambda_\alpha > 1.79$	$\lambda_\alpha > 1.55$	$\lambda_\alpha > 1.46$
Scale 2	$1.35 < \lambda_\alpha < 1.86$	$1.34 < \lambda_\alpha < 1.79$	$1.28 < \lambda_\alpha < 1.55$	$1.20 < \lambda_\alpha < 1.46$
Scale 3	$\lambda_\alpha < 1.35$	$\lambda_\alpha < 1.34$	$\lambda_\alpha < 1.28$	$\lambda_\alpha < 1.20$

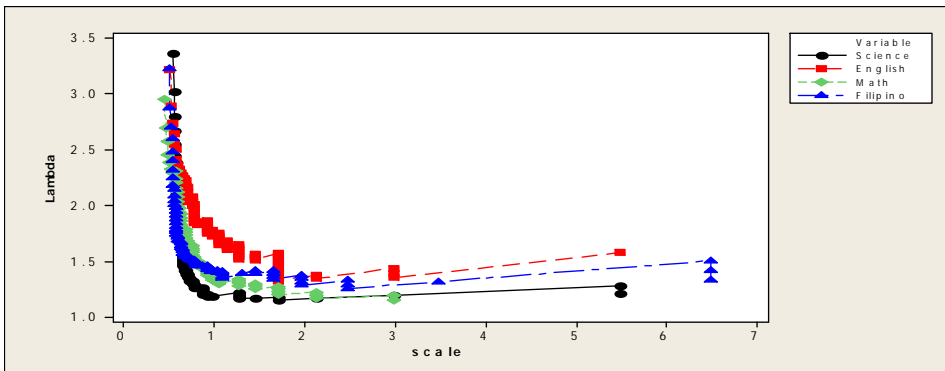


Figure 4. Superimposed spectrum of CBAPHG in four subjects.

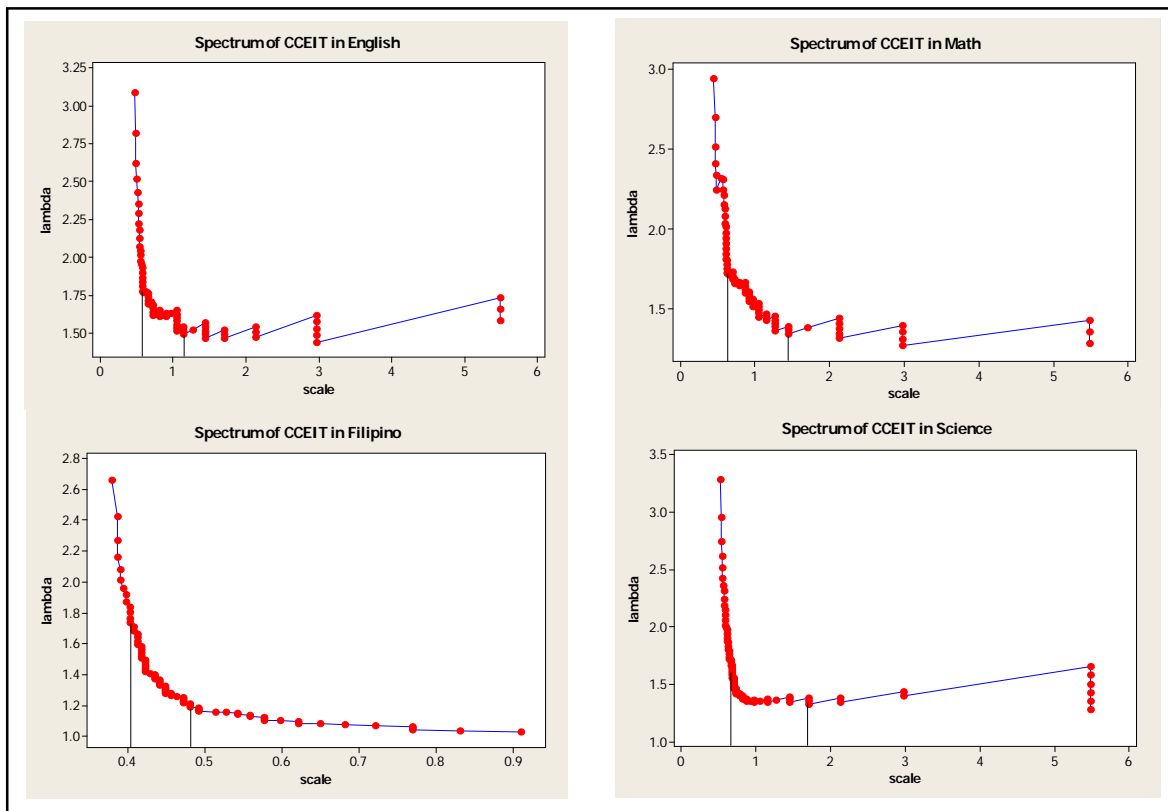


Figure 5. Spectrum of CCEIT in the four subjects.

Table 4

Scales of the Spectrum of CCEIT in Four Subjects

	English	Math	Filipino	Science
Scale 1	$\lambda_\alpha > 1.78$	$\lambda_\alpha > 1.75$	$\lambda_\alpha > 1.73$	$\lambda_\alpha > 1.60$
Scale 2	$1.51 < \lambda_\alpha < 1.78$	$1.37 < \lambda_\alpha < 1.75$	$1.21 < \lambda_\alpha < 1.73$	$1.35 < \lambda_\alpha < 1.60$
Scale 3	$\lambda_\alpha < 1.51$	$\lambda_\alpha < 1.37$	$\lambda_\alpha < 1.21$	$\lambda_\alpha < 1.35$

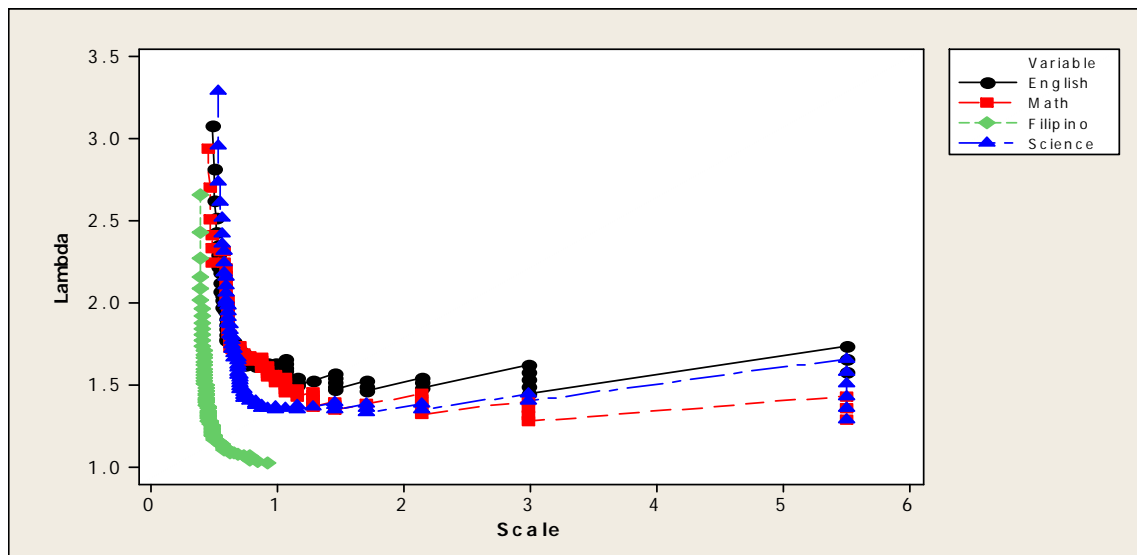


Figure 6. Superimposed spectrum of CCEIT in the four subjects.

CCEIT found English to be the most difficult among the four subjects, followed by Math, then Filipino and least difficult is in Science just like the high performing students in CBAPHG. The low-performing students who belong to the last scale have the following fractal dimensions: lower than 1.51 in English, lesser than 1.37 in Math, lesser than 1.21 in Filipino and lesser than 1.35 in Science. The superimposed graph is shown in Figure 6 to compare the variability of scores of CCEIT students in four subjects.

The spectrum of Math, Science and English fits more likely with each other than in Filipino in Figure 6. This shows that the ruggedness of

the performance of CCEIT students in either one of these three subjects: Math, Science and English, is explained better by the ruggedness of the other two. Looking at the spectrums of the last scale where the low performing CCEIT students belong, again the only subject that does not fit with the rest is Filipino, which means that the other three subjects can explain each other's ruggedness with regards to the performance of the students in this scale. It is clear also in this scale that the low performing students in this college finds Filipino to be the most difficult, followed by Math, then Science and last is English.

Table 5
Scales of the Spectrum of CTE in Four Subjects

	English	Math	Filipino	Science
Scale 1	$\lambda_\alpha > 1.78$	$\lambda_\alpha > 1.79$	$\lambda_\alpha > 2.22$	$\lambda_\alpha > 1.83$
Scale 2	$1.26 < \lambda_\alpha < 1.78$	$1.41 < \lambda_\alpha < 1.79$	$1.30 < \lambda_\alpha < 2.22$	$1.30 < \lambda_\alpha < 1.83$
Scale 3	$\lambda_\alpha < 1.26$	$\lambda_\alpha < 1.41$	$\lambda_\alpha < 1.30$	$\lambda_\alpha < 1.30$

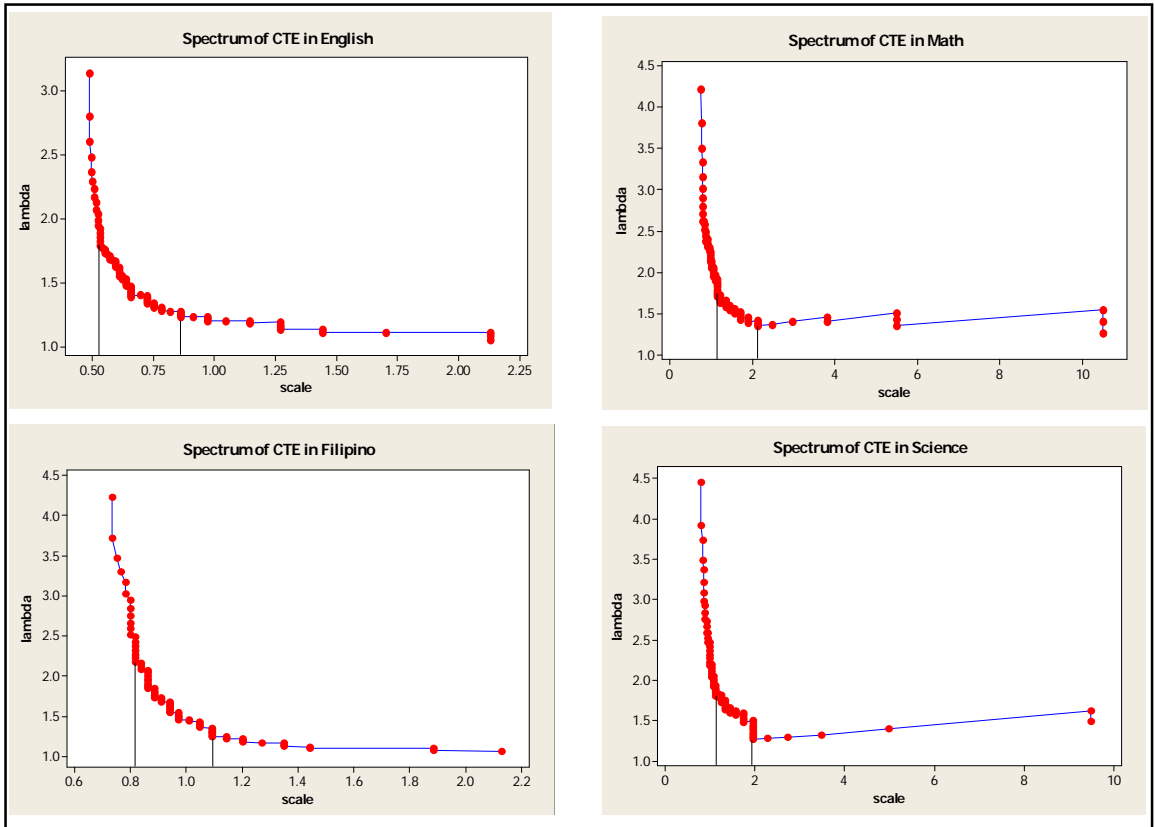


Figure 7. Spectrum of CTE in the four subjects.

Figure 7 shows that the spectrum of the performance of the students in CTE in the four subjects also has three scales. The high-performing students who belong to the first scale have greater than 2.22 fractal dimension in Filipino, greater than 1.83 fractal dimension in Science, greater than 1.79 in Math, and greater than 1.78 in English. This implies that the high performing students in CTE found both Math

and English to be the most easy among the four subjects, followed by Science, then Filipino. In English the fractal dimension of the low-performing students is less than 1.26, less than 1.41 in Math, less than 1.30 in Filipino and less than 1.30 in Science. The superimposed graph is shown below to compare the variability of scores of CTE students in four subjects.

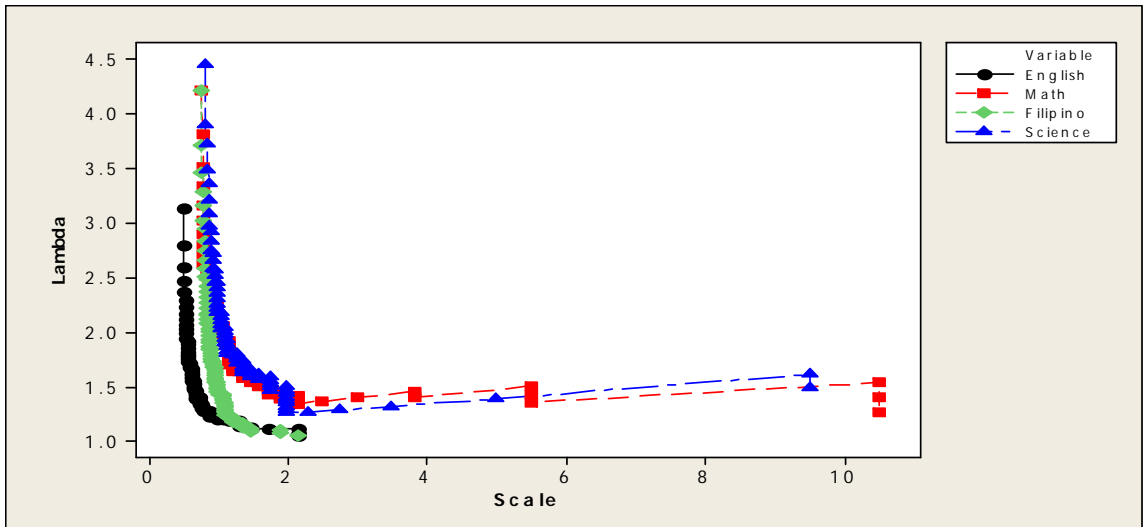


Figure 8. Superimposed spectrum of CTE in the four subjects.

As seen in Figure 8, the spectrum of Math and Science in both the high and low performing students in CTE fits more likely with each other while the other two subjects, English and Filipino, fits more likely with each other. This indicates that the ruggedness of the

performance of CTE students in both scales in each pair of subjects could be explained by the ruggedness of each other. In this scale however, the low performing students perform better in their subject in English and Filipino compared to the other two subjects.

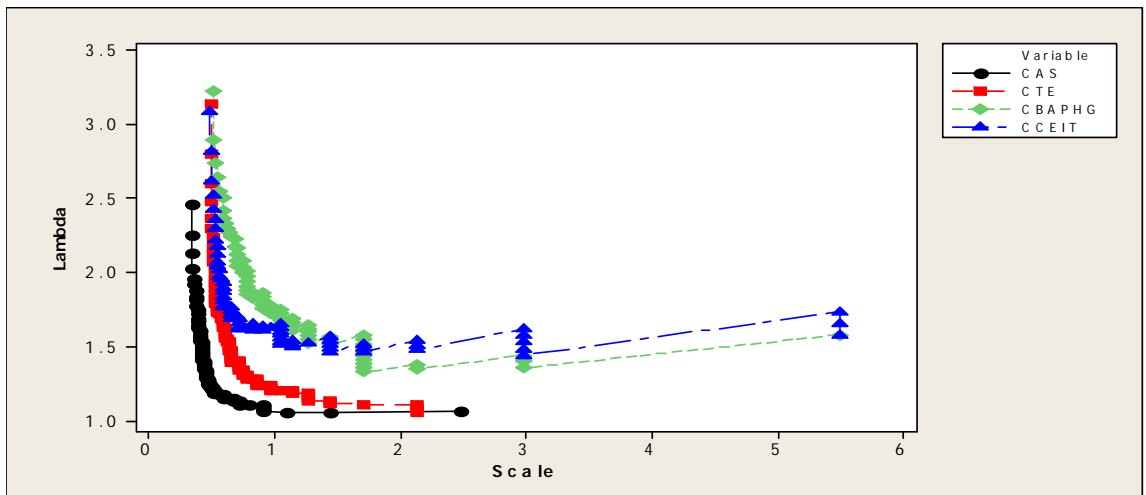


Figure 9. Superimposed spectrum of English in the four colleges.

Figure 9 shows that the academic performance of the students in English in CBAPHG and CCEIT explains the ruggedness of each other's performance since their spectrum fits most likely with each other. This

is also true for CAS and CTE. More high performing students under CAS and CTE find the subject in English to be easy than the high performing students in the CBAPHG and CCEIT. The same is true with the low

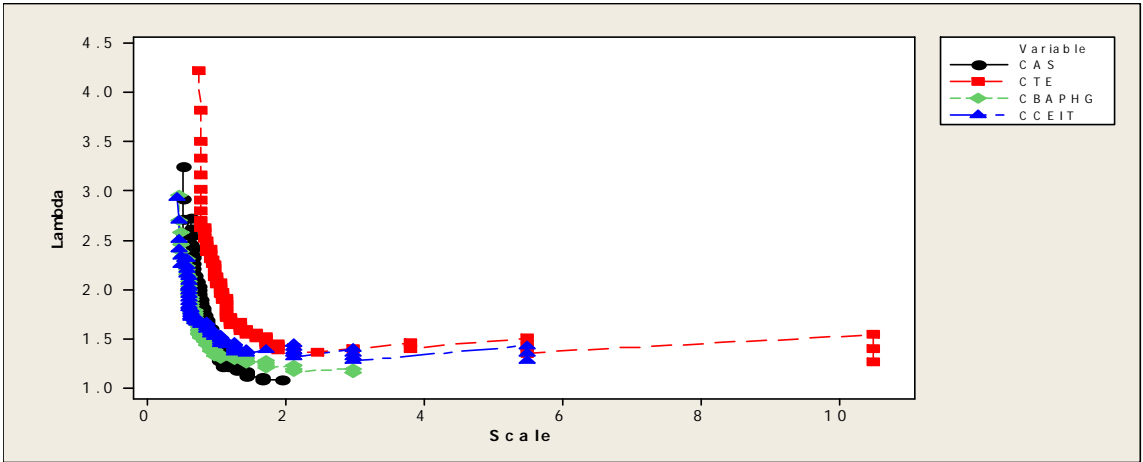


Figure 10. Superimposed spectrum of Math in the four colleges.

performing students in CAS and CTE, they find the subject easy compared to the low-performing students in the other two colleges because more students under the first scale come from CAS and CTE.

The spectrum of the academic performance in Math of the three colleges, CAS, CBAPHG and CCEIT fits more likely with each other than

the academic performance of CTE in this subject. One of these three colleges explains better the ruggedness of the other two compared to that of the performance of CTE. The students in CTE finds the subject easier compared to the other three colleges when it comes to the Math subject.

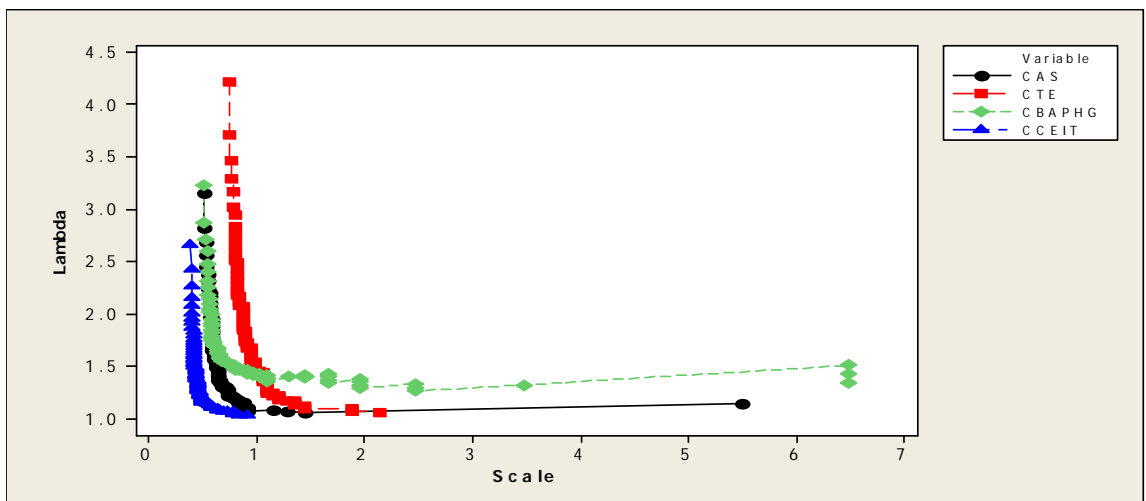


Figure 11. Superimposed spectrum of Filipino in four colleges.

Figure 11 shows that the academic performance of the students in Filipino in CAS, CBAPHG and CCEIT explains the ruggedness of each other's performance since their

spectrum fits most likely with each other compared to CTE. But it is clear that more high performing students in this subject are coming from CTE, which further implies that

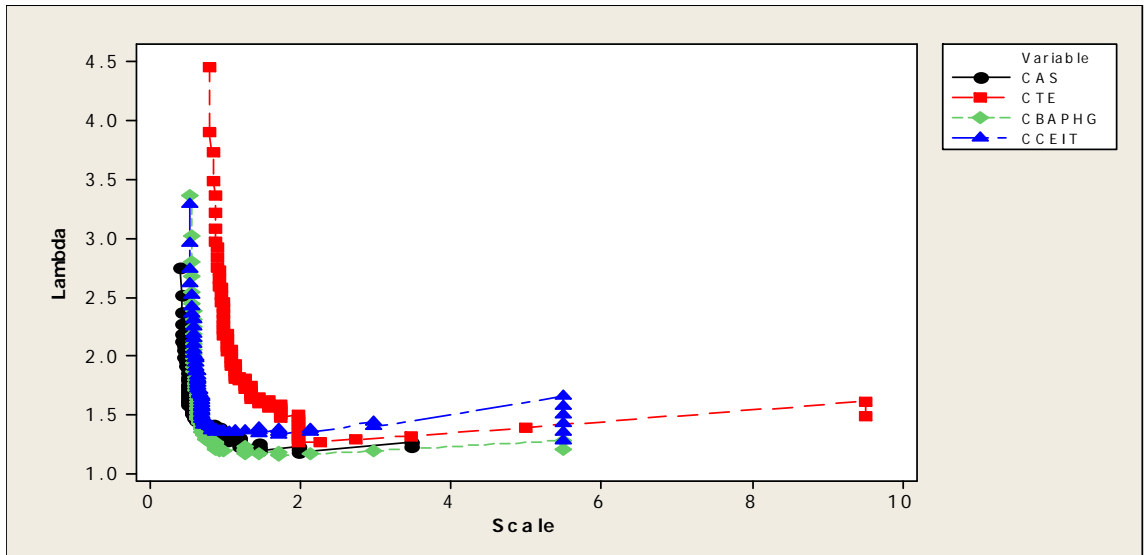


Figure 12. Superimposed spectrum of Science in the four colleges.

they performed better in this subject compared to the other three colleges.

Just like the Math subject, the spectrum of the academic performance in Science of the three colleges, CAS, CBAPHG and CCEIT fits more likely with each other than the academic performance in Science of CTE. One of the three colleges explains better the ruggedness of the other two compared to that of CTE. Again, the students in CTE finds the subject easier compared to the other three colleges.

Conclusions

Results revealed that the performance of students is fractal in nature but bounded. Results also show that there is self-similarity in students' academic performance in the four subjects Math, English, Filipino and Science across colleges in BSU. The college of CTE performed better in Math while the other three colleges CAS, CBAPHG and CCEIT found the subject to be one of the most difficult compared to English, Filipino and Science. Aside from Math, English subject was found to be very difficult for CCEIT and CBAPHG but not for CAS and CTE. The two colleges

CCEIT and CBAPHG also share commonality with regards to the subject that they find easy, both colleges performed better in Science and Filipino while this is the least subject in the college of CTE and just average subject for CAS. With this finding, implications to educators to understand and improve performance of students can be addressed. The findings could be used to smoothen the disparity of academic performance of students across colleges.

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